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Effects of coupling on turbulent gas-particle boundary layer flows at borderline volume fractions using kinetic theory

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ABSTRACT

This study is concerned with the prediction of particles' velocity in a dilute turbulent gas-solid boundary layer flow using a fully Eulerian two-fluid model. The closures required for equations describing the particulate phase are derived from the kinetic theory of granular flows. Gas phase turbulence is modeled by one-equation model and solid phase turbulence by MLH theory. Results of one-way and two-way coupled approaches are compared with the available experimental and numerical results. Results show that one-way coupled approach is more efficient for particulate velocity prediction in dilute flows. But, if the gas-phase flow characteristics are desired, the two-way coupled approach should be used. Effects of free stream velocity on the coupling are discussed.

1. Introduction

Gas-particle flows are frequently encountered in the natural environment and industrial processes, such as desert and sand storms, pollutions in industrial or urban regions, fluidized bed and pneumatic conveying of particles [1]. There are many parameters which determine the gas-particle flow characteristics; for example, particle mass loading rate ρp (Tsuji and Morikawa [2]), gravity g (Taniere et al. [3]), inter-particle spacing (Sato et al. [4]), particle volume fraction α_p (Elghobashi [5]), velocity gradient of the flow (Li et al. [6]).

Elghobashi [5] classified gas-solid flows when the solid volume fraction is less (dilute) or greater (dense) than 0.1%. When the flow is dilute, effect of presence of particle on the carrier fluid is negligible. The interaction in this regime is named one-way coupling. Also, a homogeneous model can be used in this regime same as the investigation done by Mirzaei and Dehghan [7]. When the flow is dense, the momentum transfer from particles to the carrier phase is large enough to modify the characteristics of the flow, and the interaction in this regime should be considered. This consideration could be done by using two-way coupling. Inter-particle and wall-particle collisions may be important in dense flows and considering the collisions leads the four-way coupling. But, this classification is questionable in borderline values of the solid volume fractions. In this study, it is desired to see which way of coupling is efficient for engineering purposes.

Giacinto et al. [8] studied the coupling effects for the first time. They investigated the behavior of particles based on a one-way model and a two-way model approaches. Also, they proposed a correlation for pressure drop arising from the particulate phase. Nasr and Ahmadi [9] studied the turbulence modulation due to its interaction with dispersed solid particles in a downward fully developed channel flow. They used the Eulerian framework for the gas-phase, whereas the Lagrangian approach was used for the particle-phase. The effect of turbulence on the flow-field was included via the standard k–ε model. Nasr et al. [10] studied the effects of

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particle–particle collisions and the two-way coupling on the dispersed and carrier phase turbulence fluctuations in a channel flow based on a DNS solution. They found when the particle collisions were neglected but the particle–fluid two-way coupling effects were accounted for, the two-way coupling and the particle normal fluctuating velocity decreased near the wall causing a decrease in the particle deposition velocity. In the case of the four-way coupling in which both inter-particle collisions and two-way coupling effects were present, it was found that the particle deposition velocity increased compared with the one-way coupling case. When the particle aerodynamic interactions were added to the four-way coupled case (termed six-way coupled case), no significant changes in the mean fluid and particle velocities and the fluid and particle fluctuating velocities were obtained.

Researchers investigate particles in particulate flows using two major approaches: Lagrangian and Eulerian. Lagrangian model traces the path of individual particles on their way through the flow field. This needs long computational time in comparison with Eulerian method. Eulerian approach is more efficient for engineering purpose but closure models are the shortcoming of the Eulerian approaches [11]. Closure relations for particulate flow have been derived from kinetic theory of granular flow in this study.

Similarities between non-uniform dense gases and particulate flows were a start point for developing the kinetic theory of granular flows. The kinetic theory of granular flow allows determination of pressure and viscosity of the solid particles by incorporating the flow, gas, and particle characteristics and properties [12, 13]. Lun et al. [14] applied the kinetic theory of granular flow for the first time. This model has been used and developed by researchers such as Sinclair and Jackson [15], Ding and and Gidaspow [16], Huilin et al. [13], Vejahati et al. [17], Dehghan and Basirat Tabrizi [18], and Yusof et al. [19].

Taniere et al. [3], Slater et al. [11], Wang and Levy [20, 21], and Dehghan and Basirat Tabrizi [18, 22] studied the turbulent boundary layer gas-particle flows. Taniere et al. [3] concentrated on the particle response to fluid turbulence in a dilute horizontal boundary layer. Slater et al. [11] used a one-way coupled method to predict the deposition rate of particles in the dilute region. Wang and Levy [20, 21] studied particle motion in a vertical boundary layer experimentally and numerically. They simulated the flow using the commercial ANSYS CFX-4 software based on the kinetic theory of granular flow. Dehghan and Basirat Tabrizi [18] studied on the modeling of a dilute turbulent gas-particle flows near the solid flat wall. They introduced a new inlet condition for the granular temperature balance equation. Also, they investigated the self similar profiles of particle velocity in the boundary layer which were independent of location and free stream velocity. Dehghan and Basirat Tabrizi [22] investigated the need of a turbulent model for a turbulent particulate phase and showed that the conventional turbulence models which are based on the conventional fluids are not suitable for particulate flows. However, they indicated that the results of a turbulent model for the particulate flow are more precise in comparison with a laminar model, especially near the wall.

In this study, one-way and two-way coupled approaches have been used to predict the experimental velocities of Wang and Levy [20, 21]. Motion of particles with 60 μm diameter (d_p) and 1680 kg/m^3 material density (ρ_{mp}) at 0.6 and 1.0 kg/m^3 particulate loading rates (ρ_p) and 20 and 30 m/s free stream velocities near a flat solid boundary was simulated numerically. These quantities were used on the basis of experiments of Wang and Levy [20, 21]. So, the volume fraction of particulate phase is in the borderline value. Eulerian-Eulerian two-fluid model was used to model the flow. Closure relations for particulate phase have been derived from the kinetic theory of granular flow. Effects of free stream velocity and way of coupling on the velocity of gas and particulate phases are investigated and the numerical simulations are compared with the available numerical and experimental data of Wang and Levy [20, 21].

2. Analysis and Modeling

For two-dimensional turbulent gas flow in a boundary layer of a solid flat plate (figure 1), one can write:

$$U = u + u' \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

$$u_i \frac{\partial u_1}{\partial x_i} = -\frac{1}{\rho} \nabla . \tau \tag{3}$$

U, u, and u' are instantaneous, mean, and fluctuating values, respectively. “1” is the upstream flow direction (the tangential direction to the plate). τ is the total stress tensor which is defined for incompressible fluids in the boundary layer of flat solid boundaries as:

$$\frac{1}{\rho} \tau = \left(\nu + \nu_t \right) \frac{\partial u_1}{\partial y} \tag{4}$$

ν_t is the turbulent (eddy) viscosity. One-equation model has been used for the gas-phase to model it [20]:

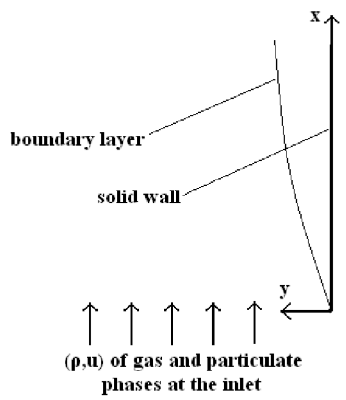

Figure 1. Schematic diagram of the problem

$$u_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}(\nu_t \frac{\partial k}{\partial x_i}) + \nu_t (\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}) \frac{\partial u_i}{\partial x_j} - 0.09 \frac{k^{1.5}}{l} \quad (5)$$

$$l = 0.54 l_m \quad , \quad \nu_t = l \times k^{0.5} \quad (6)$$

Here, k is turbulent kinetic, l denotes characteristic length of eddies which have the maximum portion of turbulent kinetic energy and l_m is the mixing length (Mixing Length Hypothesis):

$$\begin{aligned} l_m &= 0.41 y [1 - \exp(\frac{y^+}{26})] & ; \frac{y}{\delta} < 0.22 \\ l_m &= cte & ; \frac{y}{\delta} \geq 0.22 \end{aligned} \quad (7)$$

Gas phase parameters and variables are written with no subscript for convenience. Subscript 'p' denotes particulate phase. Governing equations for the particulate phase are [12]:

$$\frac{D\rho_p}{dt} = 0 \quad (8)$$

$$\rho_p \frac{DV_p}{dt} = -\nabla.\tau_p + B(u - V_p) + \rho_p g \quad (9)$$

The solid stress tensor (τ_p) is expressed in terms of the solid pressure (P_p), bulk viscosity (μ_b), and shear stress viscosity (μ_s) as [12]:

$$\tau_p = (-P_p + \mu_b \nabla V_p) I + \mu_p \{[\nabla V_p + (\nabla V_p)^T] - \frac{2}{3}(\nabla V_p) I\} \quad (10)$$

where I is unit matrix, μ_b is bulk viscosity of particulate phase, μ_p is shear viscosity of particulate phase, and B is drag coefficient which is required to couple the interface force for the two-way coupled approach. The interface momentum transfer coefficient will be as following when the porosity is greater than 0.8 [17]:

$$\begin{aligned} &\mathrm{Re} = \frac{\rho d_p |u - V_p|}{\mu} \quad ; \quad C_D = \frac{24}{\mathrm{Re}}(1 + 0.15 \mathrm{Re}^{0.687}) \\ &B = C_D \alpha^{-2.65} (\frac{3\alpha_p}{4 d_p}) \alpha \rho |u - V_p| \quad , \ (\alpha = \alpha_g \to 1) \end{aligned} \quad (11)$$

the porosity (α) represents gas-phase occupied area per the total area in any cross-section. The granular temperature (θ) is defined as a measure for the energy of the fluctuating velocity of the particles. The conservation equation of granular temperature is [12]:

$$\theta = \frac{\sum_i (u_p'^2)_i}{3} \quad (12)$$

$$\frac{3}{2} \frac{D(\rho_p \theta)}{Dt} = (-\nabla P_p I + \tau_p) : \nabla V_p + \nabla.(k_p \nabla \theta) - \gamma_p + \phi_p \quad (13)$$

Particulate phase pressure and shear viscosity have two important mechanisms in the dilute regimes, collisional and kinetic [24]:

$$\mu_p = \mu_{coll} + \mu_{kin} \quad ; \quad P_p = P_{coll} + P_{kin} \quad (14)$$

Laminar shear viscosity is [25]:

$$\begin{aligned} \mu_{kin} &= \frac{\rho_p d_p \sqrt{\theta\pi}}{6(3 - e)} [1 + 0.4 \alpha_p g_0 (1 + e)(3e - 1)] \\ \mu_{coll} &= 0.8 \rho_p d_p g_0 (1 + e) \sqrt{\frac{\theta}{\pi}} \end{aligned} \quad (15)$$

"e" is the restitution coefficient of particles and e=1 is fully elastic. In this study, "e" is set to 0.95 [25]. Pressure of the particulate phase is [12]:

$$P_p = \rho_p \theta + 2(1 + e) \alpha_p \rho_p g_0 \theta \quad (16)$$

the first term on the RHS is the kinetic and the second one is the collisional part. "g_0" is the radial distribution function [12]:

$$g_0 = [1 - (\frac{\alpha_p}{\alpha_{p,\max}})^{1/3}]^{-1} \quad (17)$$

$$\mu_b = \frac{4}{3} \alpha_p \rho_p d_p g_0 (1 + e) \sqrt{\frac{\theta}{\pi}} \quad (18)$$

$$\begin{aligned} k_p = &\frac{150 \rho_{mp} d_p \sqrt{\theta\pi}}{384(1 + e) g_0} [1 + \frac{6}{5} \alpha_p g_0 (1 + e)]^2 \\ &+ 2 \alpha_p \rho_p d_p g_0 (1 + e) \sqrt{\frac{\theta}{\pi}}] \end{aligned} \quad (19)$$

μ_b is bulk viscosity, k_p is granular diffusion coefficient, and $\alpha_{p,max}$ is the maximum particle packing limit. Granular temperature dissipation and transfer between gas and particles are expressed as [12]:

$$\gamma_p = 3(1 - e^2) g \alpha_p \rho_p g_0 \theta (\frac{4}{d_p} \sqrt{\frac{\theta}{\pi}} - \nabla.V_p) \quad (20)$$

$$\phi_p = -3B\theta \quad (21)$$

Particles in turbulent flows have two types of motion: a) Random oscillations of individual particles, measured by the classical granular temperature; b) turbulence caused by the motion of clusters of particles. These two kinds of turbulence cause two kinds of mixing, mixing on the level of a particle and mixing on the level of clusters [26]. The expressed granular temperature theory is based on the laminar motion of particles. So, for considering the cluster-like motion of particles and effects of turbulent motion of particles, a model which considers the turbulent phenomena is required [11, 22, 26, 27]. Hence, the MLH (Mixing Length Hypothesis) theory was adopted for particulate phase in which equations are the same as those in the carrier phase and are described in equations 6 and 7.

It is assumed that gas phase had no-slip condition and zero turbulent kinetic energy on the solid boundary. Free stream condition at the entrance was considered. At the exit, Neumann-type boundary condition was adopted. Particulate phase had the same boundary conditions, except on the solid wall for tangential velocity and granular temperature which are as following [15]:

$$u_{\mathrm{p,tangentioal@\ wall}} = -\frac{6 \mu_p \alpha_{p,\max}}{\pi \rho_p g_0} \frac{\partial u_p}{\partial y} \quad (22)$$

$$\theta_{\mathrm{@wall}} = -\frac{k_p \theta}{e_w} \frac{\partial u_p}{\partial y} + \frac{\sqrt{3}\pi \rho_p u_p g_0 \theta^{1.5}}{6 \alpha_{p,\max} e_w} \quad (23)$$

y-direction is normal to solid boundary, e_w is the restitution coefficient at the wall and is set to 0.75 [28]. Adopting lm at the solid wall equal to zero is not a realistic assumption for the particulate phase because of slip-velocity of particles. Hence, Neumann-type boundary condition for the mixing-length was adopted for the particulate phase at the wall [22].

These equations (1-23) describe one-way coupled approach. If one adds "*-B(u-V$_p$)*" to the RHS of the equations (3), two-way coupled approach will yield. Finite-difference method [18, 22, 29, and 30] was used to solve equations (1-23). Pseudo-transient scheme was used to decouple the equations. Central difference in space and forward in time discretization with the second order of precision was applied to be more accurate. Non-uniform structured gird (figure 2) and artificial viscosity [18, 22] were used to save time with Compaq Visual FORTRAN 6.1 software.

3. Results and Discussion

Simulation is preformed based on a 71x141 grid. Also, a 121x201 grid is used to examine mesh dependency. Figure 2 compares the simulation results in the two grids. The maximum non-dimensional difference between results of these two grids is less than 3% for both gas and solid-phases. However, computational time of fine mesh increases in several orders.

Single-phase results of the gas phase are compared with the DNS result of Spalart [31] in figure 3. "k^+" is the non- dimensional kinetic turbulent energy, "δ" is the boundary layer thickness, and "*y*" is normal direction to the wall.

The numerical result of one-equation turbulence modeling is off at the peak of the kinetic energy curve. This type of error (being out of the peak of the turbulent kinetic energy profile) is usual and originates from the modeling of turbulence phenomena instead of solving a complete and high resolution simulation (DNS). Overall, a good agreement between gas-phase simulation and the DNS can be seen.

Figure 4 shows results of one-way coupled approach in comparison with the numerical and experimental results of Wang and Levy [20, 21]. Results of current study are closer to the experiments than the previous simulation of Wang and Levy [21]. It is due to some simplification of commercial ANSYS CFX-4 code that has been used by Wang and Levy [21]. As it is discussed by Dehghan and Basirat Tabrizi [18], the accuracy of particulate phase results has a high dependency on the particulate viscosity definition and simulation. Away from the wall, drag term is dominating and velocity gradient vanishes. So, the velocity of particulates is not sensitive to the granular temperature and particulate viscosity values. The accuracy of results of gas and particulate phases has been verified.

Now, effects of coupling on the gas and particulate-phase velocities will be discussed. "2-W" in the following figures represents the two-way coupled approach.

Figures 5 and 6 show the particulate velocity results of one-way and two-way coupled approaches in comparison with experiments of Wang and Levy [21]. They could show a small discrepancy between the two

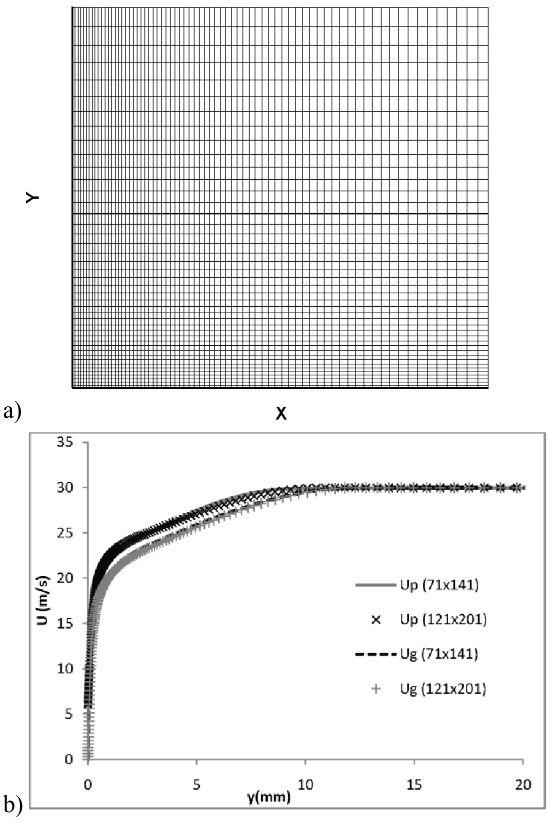

Figure 2. Used mesh (a) and mesh-dependency examination (b) (p: particulate-phase, g: gas-phase, *x*=0.5, U_∞= 30 m/s, d_p= 60 µm)

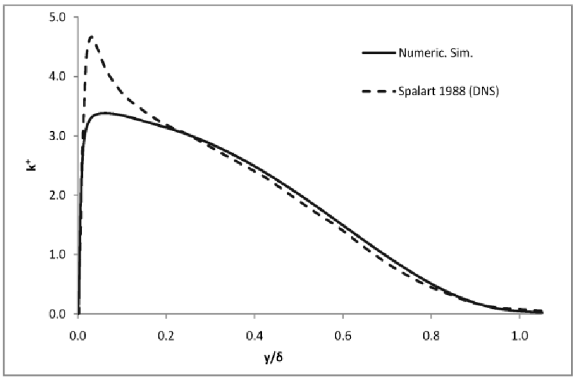

Figure 3. Gas-phase simulation in comparison with the DNS results of Spalart [27]

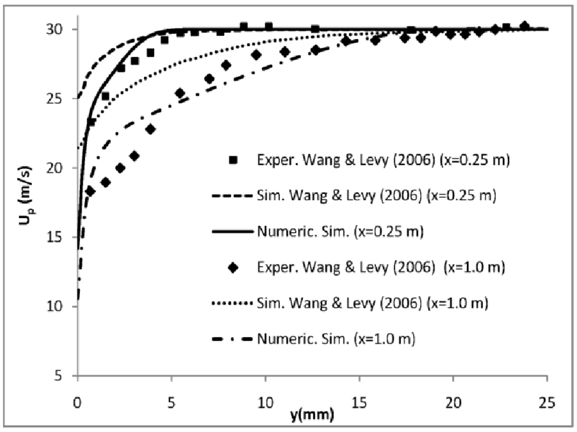

Figure 4. Examination of particulate phase simulation result accuracy (d_p=60 µm, particle loading rate ρ_p=1.0 kg/m^3, U_∞=30 m/s)

approaches. This is due to low volume fraction. Discrepancies between one-way and two-way approaches decrease by increasing the axial location (*x*-direction). Velocity results of the two-way coupled approach are higher than the one-way one. It is a result of coupling. When the two-way coupled approach is used, the simulated velocity of gas phase would increase since the particles are faster in the boundary layer and the code could consider the momentum enhancement of particles. Figures 7 and 8 are plotted for 20 m/s free stream velocity.

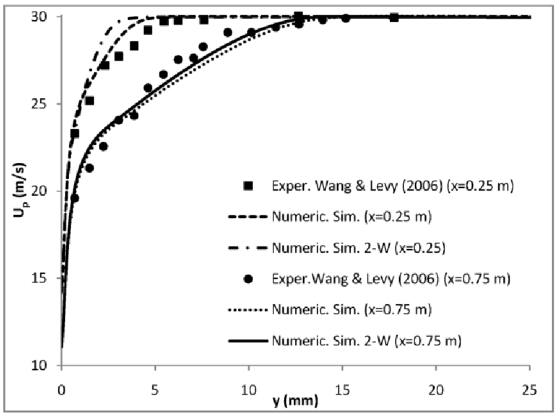

Figure 5. Results of particle velocity of one-way coupled approach in comparison with the two-way at x=0.25 and 0.75 m (ρ_p=1.0 kg/m^3, U_∞=30 m/s)

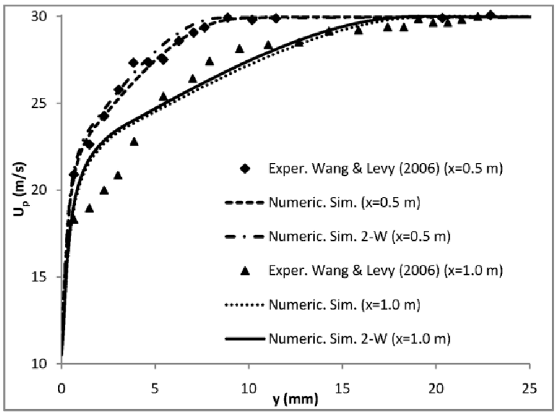

Figure 6. Results of particle velocity of one-way coupled approach in comparison with the two-way at x=0.5 and 1.0 m (ρ_p=1.0 kg/m^3, U_∞=30 m/s)

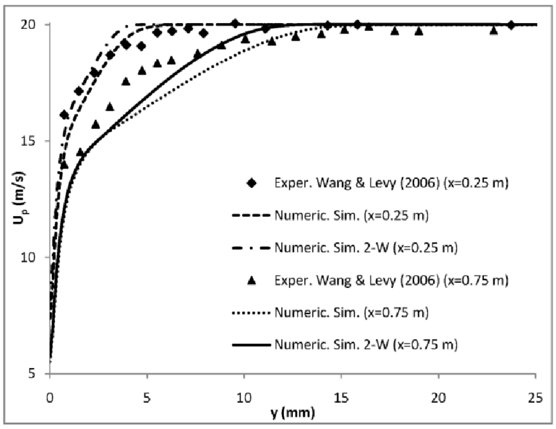

Figure 7. Results of particle velocity of one-way coupled approach in comparison with the two-way at x=0.25 and 0.75 m (ρ_p=1.0 kg/m^3, U_∞=20 m/s)

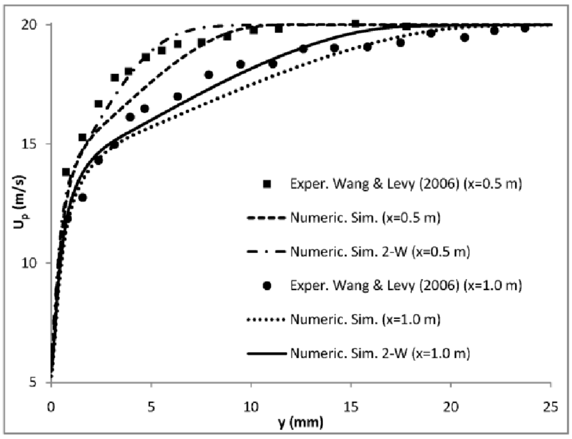

Figure 8. Results of particle velocity of one-way coupled approach in comparison with the two-way at x=0.5 and 1.0 m (ρ_p=1.0 kg/m^3, U_∞=20 m/s)

Again, it can be seen that two-way coupled approach has still little effects on the results. However, these effects are more sensible than the 30 m/s free stream velocity. By decreasing the free stream velocity, boundary layer will grow. This growth simultaneously occurs with velocity decrease, which decreases the order of viscous term in the Navier-Stokes equations for the particulate phase. Therefore, importance of viscous term will decrease in comparison with the drag term in the equation of motion. On the other hand, two-way coupled approach has a drag nature. In other words, when drag has a greater order of magnitude, the two-way coupled approach will be more important. Furthermore, as it was expected the results of the two-way coupled approach are closer to the experiments. An important parameter for engineering purposes is the computational time. The cost of the two-way coupled approach is very high in comparison with the achieved accuracy. In this study, simulation of a one-way coupled model lasts less than 72 hours for non-dimensional residuals lower than 0.001 on a PC with 3.12 GHz CPU and 4 GB RAM. But, the two-way coupled model needs more than 96 hours to achieve the same residuals of the one-way coupled model. Another matter arising from the two-way coupled approach is the tendency of the code to diverge. So, the code needs finer meshes and further numerical efforts to achieve the same residual of the one-way coupling.

From the presented discussion and results of the one-way and two-way models, one can conclude that the one-way coupled approach is more advantageous for engineering purposes in prediction particles velocity in such dilute conditions. Nevertheless, is it true for the carrying phase?

Figure 9 presents effects of the particulate loading rates (ρ_p= 1.0 and 0.6 kg/m^3) on the velocity of the carrier gas phase. It shows that particulate loading can modify the gas-phase velocity profile. To see effects of particulate phase on carrier phase more obviously, figure 8 has been drawn. Effect of particulate loading rate on the wall friction factor is not negligible even in such dilute conditions as it can be seen in figure 10. Friction factor in 1.0 kg/m^3 and 0.6 kg/m^3 loading rates increases 12% and 8% respectively in comparison with the clean gas flow. It is due to some simultaneous effects which affect gas-phase flow characteristics. From figure 8 one can see that

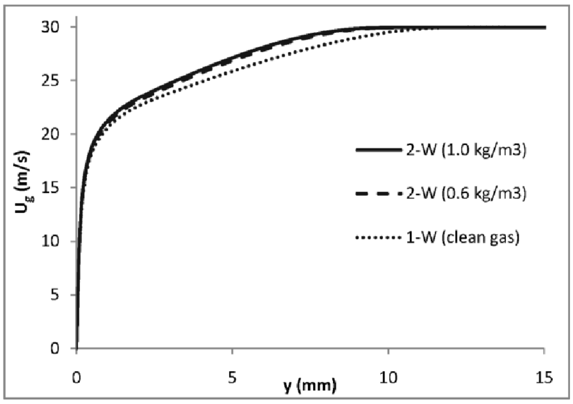

Figure 9. Effects of particulate loading rate on the gas-phase velocity in the boundary layer (ρ_p=0.0, 0.6, and1.0 kg/m^3, U_∞=30 m/s)

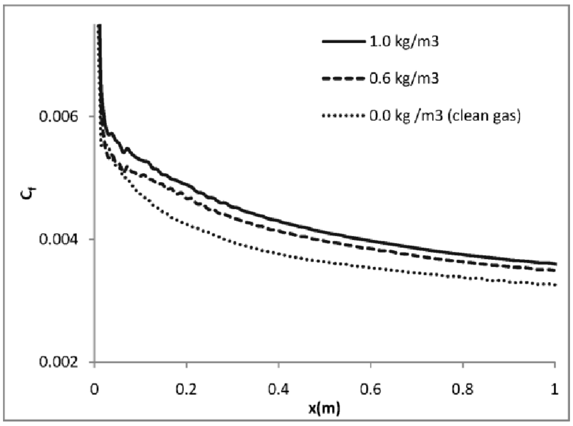

Figure 10. Effects of particulate loading rate on the wall friction factor (ρ_p=0.0, 0.6, and1.0 kg/m^3, U_∞=30 m/s)

the gas-phase velocity increases due to the particle existence. Simultaneously, boundary layer thickness attenuation occurs. These two simultaneous effects beside the high particle slip-velocity, which can be seen from figures 5-8, influence the gas-phase velocity more considerable. Therefore, the two-way coupled approach could be important in dealing with carrier gas-phase; while, it is not efficient for the particulate phase predictions for engineering purposes in such dilute flows.

4. Conclusion

Dilute turbulent gas-solid boundary layer flow has been studied numerically. Eulerian two-phase model based on the kinetic theory of granular flow has been used to model the flow. Turbulence of gas and solid phases has been modeled by the "*k-l*" and MLH models. Collisions have been considered via restitution coefficients (e and e_w). Costs and benefits of the way of coupling (one-way / two-way coupled approach) have been investigated on the simulations. Highlights of this study may be stated as following:

- One-way approach is more advantageous among engineering purposes for particulate velocity predictions.
- For the carrier phase, change in flow characteristics is considerable in comparison with the change of particulate velocity when the two-way coupled approach is used. So, when the gas-phase flow characteristics are needed, the two-way coupled approach should be applied.
- At lower free stream velocity, the two-way coupled approach is more efficient for particulate phase simulations.
- Boundary layer thickness attenuation, gas-phase velocity increase in the boundary layer, and high particle slip-velocity affect gas flow particularly near the wall.

To summarize, two-way coupled approach is not advantageous for such dilute flows, even in borderline volume fraction values (volume fractions near 0.1%). This is due to the computational costs and amount of achieved accuracy in the results. One-way coupled approach results are accurate enough for engineering purposes. For example in the current simulation, the two-way coupled approach needs up to 50% more computational time than the one-way for a same error limit and mesh size.

Nomenclature

C	drag coefficient
d	diameter (m)
e	Restitution coefficient
I	unit matrix
K	turbulent kinetic energy
l	length scale
P	pressure
Re	Reynolds number
t	time
u, v	velocity
x, y	coordinate
Greek symbols	
α	volume fraction
β	momentum transfer coefficient
γ	granular temperature dissipation
δ	boundary layer thickness
Θ	granular temperature
υ	kinematic viscosity
τ	stress tensor
μ	dynamic viscosity (Pa.s)
ρ	density (kgm-3)
Φ	energy transfer between phases
subscripts	
$coll$	collision
D	drag
g	gas
i, j	indices
kin	kinetic
m	mixing-length

mp material density

p particulate phase

t turbulent

w wall

Superscripts

' fluctuating value

+ non-dimension value

T transpose

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