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Numerical simulation of Laminar Free Convection Heat Transfer around Isothermal Concave and Convex Body Shapes

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ABSTRACT

In the present research, free convection heat transfer from isothermal concave and convex body shapes is studied numerically. The body shapes investigated here, are bi-sphere, cylinder, prolate and cylinder with hemispherical ends; besides, they have the same height over width (H/D = 2). A Numerical simulation is implemented to obtain heat transfer and fluid flow from all of the geometries in a wide range of Rayleigh numbers. The results show that flatness, concavity and smoothness have major effects on estimation of free convection heat transfer. As the total surface heat transfer area changed by altering the geometry, the local Nusselt number are compared for these body shapes; as well; it shows that concave surfaces has adverse influence on transferring heat. In addition, the current results reveal the average Nusselt numbers based on square surface area are not affected by the geometries for the laminar range of Rayleigh numbers. Besides, "incompressible ideal gas model" is used for the variation of density in free convection heat transfer. This model has the capability to be utilized in the cases with high temperature differences between the fluid and the bodies' surfaces.

1. Introduction

Investigation of convection heat transfer from diverse body shapes has been the subject of many researches over some decades. Among various mechanisms, free convection heat transfer has great significance as a result of its applications in several industrial sectors. The importance of this heat transfer phenomenon is its low cost of implementation and natural occurrence. However, free convective flows are complex to control because they depend on different parameters such as the thermo-physical properties of the fluid and the geometry of the body shape.

One important category of problems usually encountered in engineering is calculation of free convection heat transfer from a single immersed body shape. Three geometries of a sphere, a very long horizontal cylinder and a vertical cylindrical surface are frequently encountered in diverse technological units. Hence, they have been the subject of many numerical, analytical and experimental investigations as reviewed by Martynenko and Khramtsov [1].

Free convection heat transfer from short cylinders with active ends, cones, cuboids and other bodies with different finite aspect ratios have also been studied previously. Chamberlain [2] experimentally obtained heat transfer from a sphere, cube positioned in three orientations and a vertically aligned bi-sphere.

Later, Sparrow and Stretton [3] proposed an expression to calculate natural convection heat

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transfer from some bodies of unit aspect ratio such as a cube in different orientations. In addition, Yovanovich and Jafarpur provided analytical models for laminar natural convection from horizontal and vertical isothermal cuboids for a wide range of Prandtl numbers [4]. Besides, an analytical model with numerical simulations was proposed by Eslami et al. [5] to calculate laminar natural convection heat transfer from isothermal cones of arbitrary aspect ratio and orientation (tip upward, tip downward and horizontal) with active end for all values of Prandtl number. Besides, Eslami and Jafarpur [6] analytically studied finite isothermal cylinders of arbitrary orientation and aspect ratio (horizontal, vertical and inclined) with active ends.

In addition to the large number of correlations available for various body shapes, free convection heat transfer from an immersed body can be calculated by using the procedures provided for arbitrary geometries [7-11]. The square root of the active heat transfer surface area (√A) as the characteristic length and a general expression for Nusselt number was provided by Yovanovich [7]. The expression contains a parameter, Body Gravity Function (BGF), which accounts for the geometry effects; besides, he presented numerical values of BGF for various body shapes by using experimental data. Because of the significance of this parameter on estimation of heat transfer, Lee et al. [9] and Jafarpur [10] proposed analytical expressions to calculate the value of this function. Recently, Eslami and Jafarpur [11] have used a novel model by introducing the dynamic body gravity function, to improve the accuracy of the analytical method.

Although, in the publications mentioned above, the powerful tools were utilized for prediction of laminar free convection heat transfer, they are mostly applicable to convex bodies. This is mainly because the analytical methods are based on boundary layer assumptions which might not be appropriate for all concave surfaces; for instance, the flow may become stagnant or circulated adjacent to concave body surfaces resulting in a reduction in the rate of heat transfer.

Little attention, unfortunately, has been paid to compare free convection heat transfer from concave and convex body shapes. Among the publications conducted to free convection heat transfer, the slightly concave body shapes could be found such as wavy walls [12-18], wavy cones [19-22] and a horizontal upward hemispherical cavity [23].

Therefore, the objective of the present study is to investigate the effect of flatness, concavity and smoothness on estimation of free convection heat transfer from four different body shapes with the same height over width (H/D=2). Moreover, the amount of total heat transfer is compared and discussed for all of the geometries. Besides, a numerical method is proposed in order to model free convection heat transfer and this model has the capability to be utilized in the cases with low and high temperature differences between the fluid and the bodies' surfaces.

2. Problem definition

The geometries of the four body shapes that are considered in the present research are shown in Fig. 1. It is assumed that the geometries are suspended in a large space filled with incompressible and viscous air. The surrounding quiescent fluid (air) is at a uniform temperature T_0, while the bodies' surfaces are heated at a constant temperature T_w. Hence, the mechanism of heat transfer is free convection. The present results are based on the following assumptions:

- The flow is axi-symmetric with respect to the vertical axis that is parallel to the line of gravity.
- The fluid flow is steady state and laminar.
- Radiation effects and viscous dissipation are neglected.
- All the fluid properties are taken to be constant except for the variation of density with temperature in the buoyancy term.

Therefore, the following equations of conservation of mass (continuity equation), momentum and energy should be solved in the axi-symmetric coordinate system:

Continuity equation:

$$\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}=0 \tag{1}$$

Momentum equations:

$$V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{{V_\theta}^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+ \vartheta\left[\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)V_r-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right]-g\cos(\theta) \tag{2}$$

$$V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_r V_\theta}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+ \vartheta\left[\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)V_\theta+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right]-g\sin(\theta) \tag{3}$$

Energy equation:

$$V_r\frac{\partial T}{\partial r}+\frac{V_\theta}{r}\frac{\partial T}{\partial \theta}=\kappa\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)T \tag{4}$$

where v_θ and v_r are the azimuthal and radial velocity components.

- $T=T_w$ and no slip boundary condition at the body surface.

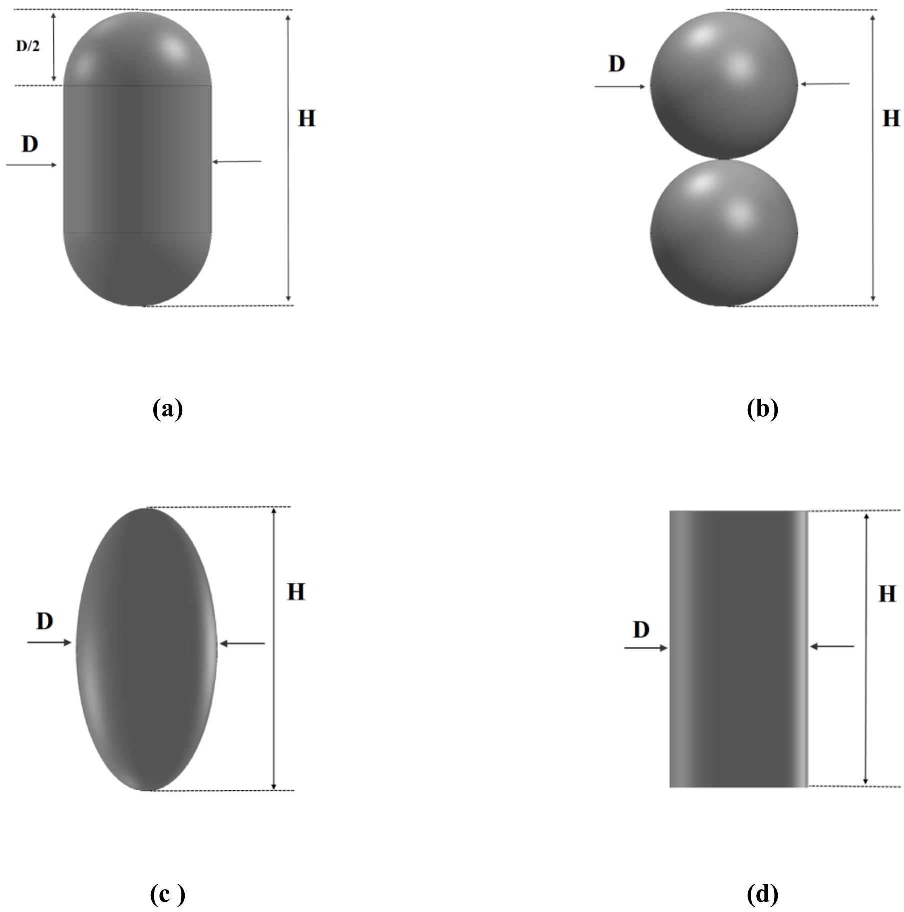

Figure 1. Geometry of different body shapes in the present study; a) Cylinder with hemispherical ends, b) Bi-sphere, c) Prolate, d) Cylinder

The incompressible ideal gas model for calculating density variation is appropriate in free convection because the density of the fluid (air) around the bodies vary only with temperature; in other words, the temperature does not vary with air pressure. Furthermore, this model has the capacity to be used in a wide range of temperature differences particularly high temperature one like 1000 ^{o}C ($T_S - T_\infty = 1000^0 C$).

$$\rho = \frac{P_{op}}{\frac{R}{M_w} T} \quad (5)$$

When P_{op} is the operating pressure and it is set to 101325 Pa.

The above governing equations (1)–(4) have to be solved according to the following boundary conditions:

- Symmetry conditions on the axis line for velocity and temperature.
- $T=T_0$ and $P = P_0$ at the external boundary of the computational domain (see Fig. 2).

3. Numerical solution procedure

The free convection problem discussed in the previous section is solved by using finite-volume method with a segregated multi-grid solver. Fig. 2 shows the configuration of computational domain and the boundary conditions for one of the geometries. The momentum equations are discretized by the body-force-weighted scheme and the power law scheme is also used for pressure and energy equations [24]. The Semi Implicit Method for Pressure Linked Equations (SIMPLE) of Patankar [24] is used to couple the pressure and velocity in momentum equations. Besides, the under-relaxation parameters are set to 0.3 for pressure, 1 each for density, body forces and energy and 0.7 for momentum equations. Moreover, thermo-physical properties of the fluid (air) are evaluated at film temperature except the density of air that is obtained from the incompressible ideal gas model. A grid of

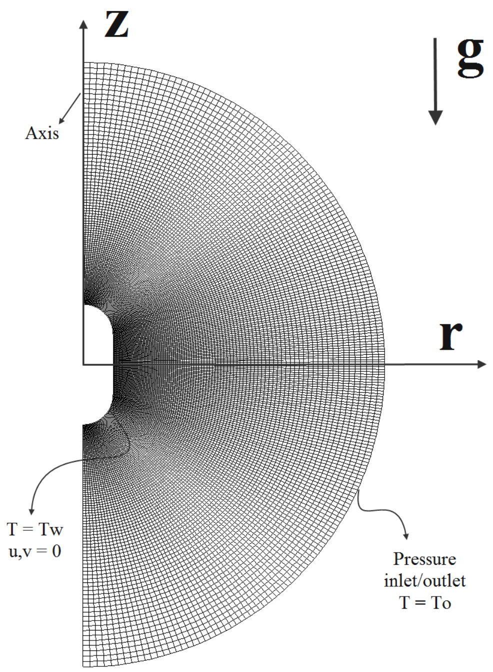

Figure 2. Computational domain and boundary conditions

rectangular cells with fine stretch near the wall is used (Fig. 2). Besides, the numerical simulations for all of the geometries are performed with different numbers of cells as shown in Fig. 3. To put differently, this figure shows that the numbers of cells used in the computational domains for all of the body shapes are enough to obtain grid-independent solutions. In order to gain a converged solution, iterations in the point implicit (Gauss Seidel) solver in conjunction with an algebraic multi-grid method are continued until the sum of normalized residues become less than 10^{-4} for momentum and continuity equations and 10^{-7} for energy one.

The above simulation method is first applied to an isothermal bi-sphere and a cylinder with hemispherical ends to validate the accuracy of the procedure. The numerical results are compared with the available experimental data of Jafarpur [10] and Hassani [25]. Fig. 4 and 5 illustrate there are excellent agreements between the results of present study and the experimental data in a wide range of Rayleigh numbers ($0 < Ra_{\sqrt{A}} < 10^8$). Therefore, the reliability of the present numerical method is well done and this method can be applied to other geometries where there is no available experimental data.

4. Results and discussions

All bodies shown in Fig. 1 are respective to each other and have the same height over width (H/D = 2). They are made by removing some part of a

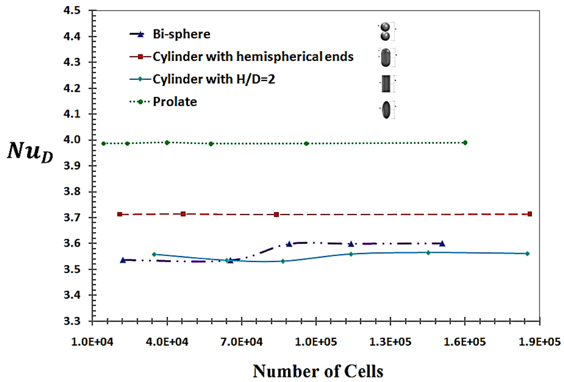

Figure 3. Effect of grid resolution on free convection heat transfer from at $Ra_D = 1.76 \times 10^3$

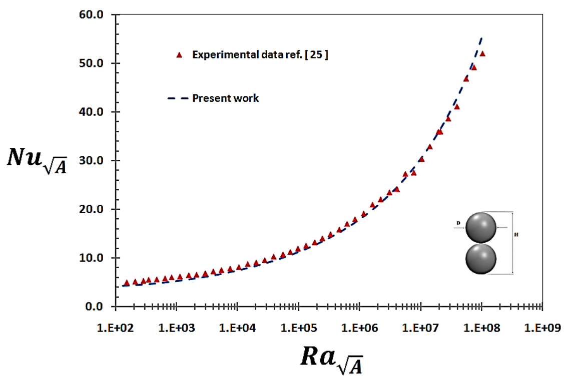

Figure 4. Comparison of the present numerical results for a bi-sphere with experimental data of Jafarpur [25]

cylinder with H/D = 2 (Fig. 1d). For instance, a cylinder with hemispherical ends (Fig. 1a) is similar to the cylinder by removing its sharp edges; in other words, it contains a cylinder with H/D = 1 and two distinct hemisphere located at the top and bottom of that cylinder. Besides, bi-sphere (Fig. 1b) is an example of concave body shapes and the rest of the bodies are examples of convex ones.

Moreover, the comparison between the results of the cylinder with hemispherical ends, prolate and the cylinder (Figs. 1a, 1c & 1d) provides the opportunity to investigate the effects of flatness and smoothness of surfaces on estimating free convection heat transfer. In addition, the comparison of local heat transfer from the bi-sphere and the cylinder with hemispherical ends (Figs. 1b & 1a) reveals the influence of concave surfaces on evaluating this kind of heat transfer.

In this section, free convection heat transfer from the body shapes are discussed based on results of numerical simulations.

Fig. 6 shows contours of temperature distribution around the four body shapes for

$Ra_D=10^6$. The temperature distributions near the bottom and top parts of the body shapes under

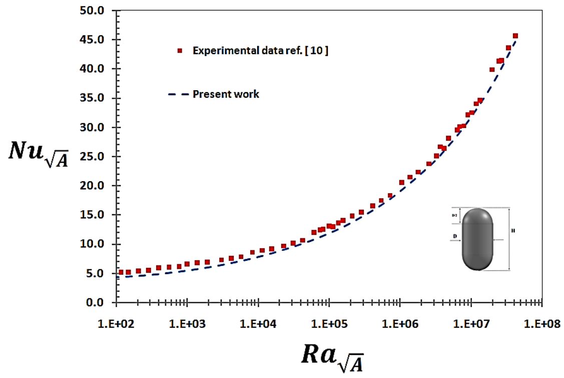

Figure 5. Comparison of the present numerical results for a cylinder with hemispherical ends with experimental data of Jafarpur [10]

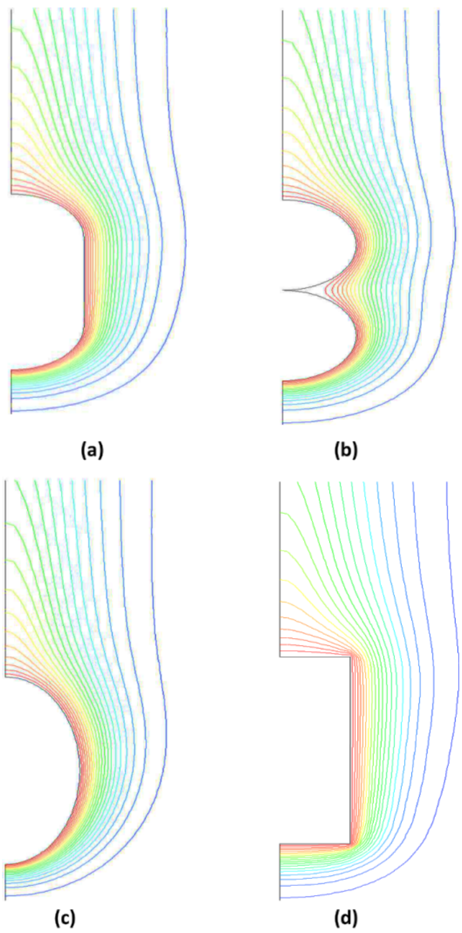

Figure 6. Isothermal contours around the bodies' surfaces; a) Cylinder with hemispherical ends, b) Bi-sphere, c) Prolate, d) Cylinder

consideration are almost the same. Besides temperature behaviors for the middle parts of the convex bodies are approximately the same; however, for the bi-sphere as a concave body shape, a very different temperature behavior is observed in the concave surfaces located in the middle part of this geometry. To gain a better understanding, Fig. 7 is plotted to compare the local convection heat transfer coefficient for the bodies under investigation. The general observed trend is that the lower parts of the bodies are associated with higher values of heat transfer coefficient as a result of having thinner thermal boundary layer. Also, heat flux decreases relatively in the plume on the top part. However, the interesting behavior is observed in the middle part. The concave surfaces of the bi-sphere are responsible for the decrease in local heat transfer especially near the corner of the bi-sphere's concave surfaces.

The outcome is that free convection heat transfer from the middle of the concave shape is relatively low in comparison with the convex parts of the body shape; hence, the concave part is not contributing well to the heat transfer.

Fig. 8 illustrates the streamlines near the surface of all body shapes shown in Fig. 1. It illuminate that the flow cannot easily pass the concave surfaces and it is a major cause of decreasing the rate of heat transfer near the concave surfaces; generally, it can be said there exists a low velocity zone near the concave surfaces (the middle of the bi-sphere).

In addition, Fig. 9 shows the dependence of the average $\overline{Nu}_D$ on Ra_D in the whole laminar range of fluid flow for the geometries under investigation. The comparison between the results of the bodies illustrates that in a wide range of Rayleigh number, the prolate has the best rate of heat transfer performance among the rest of bodies (shown in Fig. 1). This is mainly because this geometry has the smoothest surfaces and the fluid flow could pass over the surfaces much easier. Besides, the cylinder with hemispherical ends has also a higher average Nusselt number in comparison with the bi-sphere and the cylinder.

In addition, cylinder and bi-sphere have approximately the same average heat transfer coefficient for most values of Rayleigh number. The results illustrate that although, concave surfaces (the middle part of bi-sphere) may cause to increase total surface area of a concave body, they can contribute to decreasing total free convection heat transfer. Besides, the fluid flow could not pass the cylinder's surfaces as easy as the other convex body shape under investigation.

Fig. 10 reveals that the average Nusselt numbers based on square surface area ($\overline{Nu}_{\sqrt{A}}$) are not affected by the geometries for a wide range of Rayleigh numbers. In other words, it proofs that in a laminar range of Rayleigh number, the average nusselt number is approximately the same for all of the body shapes. Therefore, by using this characteristic length, it is possible to predict heat transfer from more complex shapes with low cost of calculations and time. For instance, it is possible to calculate the free convection heat transfer from a complex geometry like a human body in different

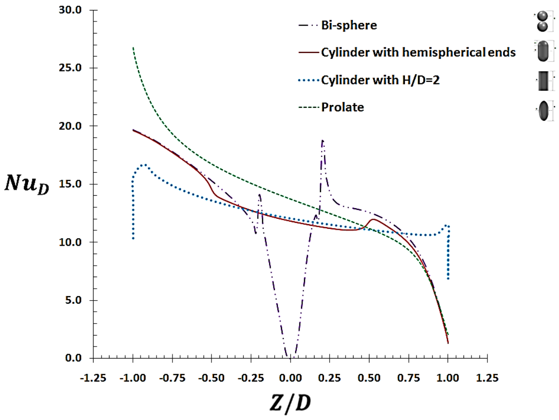

Figure 7. Distribution of local heat transfer coefficient; $Ra_D = 10^7$

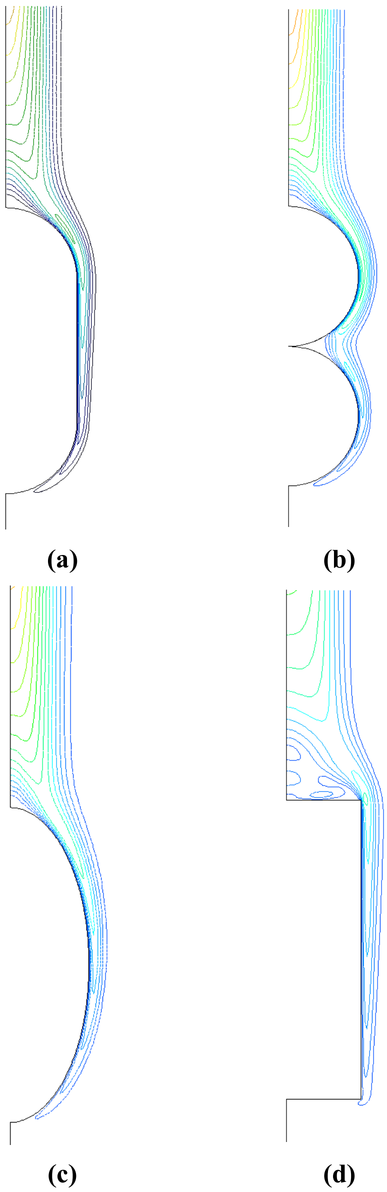

Figure 8. Streamlines near the bodies' surfaces; a) Cylinder with hemispherical ends, b) Bi-sphere, c) Prolate, d) Cylinder

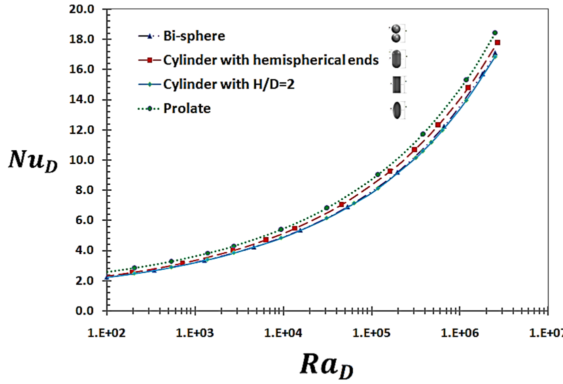

Figure 9. Comparison of $\overline{Nu}_D$ for all body shapes in a wide range of Ra_D number

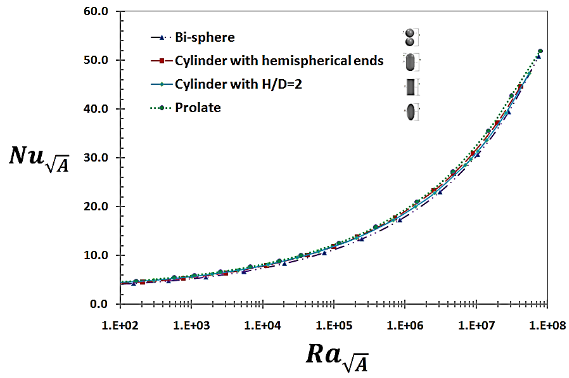

Figure 10. Comparison of $\overline{Nu}_{\sqrt{A}}$ for all body shapes in a wide range of $Ra_{\sqrt{A}}$ number

conditions only by using the temperature and surface area of the body and ambient temperature (without computational modeling of the complex body).

5. Conclusions

Numerical simulations have been performed to investigate laminar free convection heat transfer from four different isothermal concave and convex body shapes; besides, they are respective to each other and have the same height over width (H/D = 2).

It is concluded that:

- "incompressible ideal gas model" has the capability to be utilized in the cases with high and low temperature differences between the fluid and the bodies' surfaces.
- The results show that flatness, concavity and smoothness have major effects on estimation of free convection heat transfer.
- The prolate has the best rate of heat transfer performance among the rest of bodies under investigation; because this geometry has the

smoothest surfaces and the fluid flow can easily pass over them.

- The cylinder with hemispherical ends has also a higher average Nusselt number in comparison with the bi-sphere and the cylinder.
- The local heat transfer coefficient from the concave surfaces of the bi-sphere is significantly decreased especially near the corner of the bi-sphere's concave surfaces.
- The cylinder and the bi-sphere have approximately the same average heat transfer coefficient in laminar range of Rayleigh numbers.
- Average Nusselt numbers based on square surface area ($\overline{Nu}_{\sqrt{A}}$) are not affected by the geometries for a wide range of Rayleigh numbers.

Nomenclature:

A	Surface area (m^2)
C_p	Specific heat (J/kg.K)
D	Diameter of sphere (m)
g	Gravity acceleration (m/s^2)
h	Heat convection coefficient ($W/m^2.K$)
H	Height (m)
k	Thermal conductivity (W/m.K)
M_w	Molecular weight (kg/kgmol)
Nu	Local Nusselt Number ($\frac{h_{local}D}{k}$)
$\overline{Nu}$	Average Nusselt Number ($\frac{h_{avg}D}{k}$)
P	Pressure (Pa)
Pr	Prandtle Number
R	Universal Gas-law constant (8314.47 j/ kmol. K)
Ra	Rayleigh Number ($\frac{g\beta(\Delta T)D^3}{\vartheta\alpha}$)
T	Temperature (K)
u,v	Velocity components (m/s)

Greek Symbols

β	Volumetric expansion coefficient (K^{-1})
κ	Thermal diffusivity ($N.s.m^{-2}$)
ϑ	Kinematic viscosity ($m^2.s^{-1}$)
ρ	Density ($kg.m^{-3}$)
μ	Dynamic viscosity ($N.s.m^{-2}$)

Subscripts

f	Film
o	Ambient
op	Operating condition
w	Wall

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