Journal of Heat and Mass Transfer Research

Journal homepage: http://jhmtr.journals.semnan.ac.ir

Semnan University

Heat and mass transfer of nanofluid over a linear stretching surface with Viscous dissipation effect

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PAPER INFO

Paper history:
Received: 2018-08-09
Received: 2019-02-14
Accepted: 2019-06-03

Keywords:
Stretching Sheet;
Nanofluid;
Boundary Layer;
Brownian motion;
Thermophoresis;
Viscous dissipation.

ABSTRACT

The main objective of this paper is to extend the numerical investigation of boundary-layer flow of steady state, two-dimensional flow of nanofluid over a stretching surface with the impact of viscous dissipation. The ordinary differential equations are obtained by applying similarity transformation on partial differential equations. Then, the system is solved by applying the shooting techniques together with Adams-Bashforth Moulton Method. Software Fortran is used to compute the numerical results and the resulting values are indicated through graphs and tables.

DOI: 10.22075/jhmtr.2019.15419.1214

1. Introduction

The study of fluid on a stretching surface is one of the noteworthy problems discussed in the current era as it occurs in different engineering processes like extrusion, wire drawing, melt-whirling, production of glass fibre, manufacturing of rubber sheets and cooling of huge metallic plates such as an electrolyte. Choi [1] examined the enhancing thermal conductivity of fluid with nanoparticles. The different theories of heat transfer in nanofluids are discussed by Buongiorno [2]. Kuznetsov and Nield [3] inquired the convective nanofluid in vertical plate, later they extended their work for porous medium. W.A. Khan and I. Pop [4] examined the convective boundary layer flow of nano-fluid past a stretching surface. Noghrehabadi et al. [5] inspected the effect of heat over a stretching sheet with partial slip condition.

The study of fluid on a stretching sheet is one of the essential problems discussed in the current era as it occurs in different engineering processes like extrusion, wire drawing, melt-whirling, production of glass fiber, manufacturing of rubber sheets and cooling of huge metallic plates such as an electrolyte. By applying the uniform stress, the sheet bears the incompressible flow which was first scrutinized by Crane [6]. Bachok et al. [7] explored solutions for boundary layer flow of with uniform free stream. Gupta et al. [8] have analysed the impact of suction and blowing on heat and mass transfer of nanofluid in a stretching sheet.

In fluid temperature, no doubt, viscous dissipation produces a considerable ascend. This would happen because of modification in kinetic motion of fluid into thermal energy. The impact of heat transfer of viscous dissipation passing over a nonlinearly stretching sheet was investigated by Vajravelu et al. [9]. W. Ibrahim and B. Shankar [10] analysed the MHD effect for heat transfer for velocity and for the thermal and slip boundary conditions. In recent years, MHD flows of nanofluids with or without

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heat transfer problems have also been addressed by some researchers [11-19].

In this article, first the review of the research paper of W.A. Khan., I. Pop [4] is presented, and consequently focuses on the extension by taking additional effect of viscous dissipation.

2. Governing Equations

A 2 – D study convective flow of an incompressible and viscous nanofluid through a plate in porous medium has been completed. From the slot at the origin thin solid surface is extruded which is being stretched in x-direction. The stretching velocity $u_w(x) = ax$, where a is constant $(a > 0)$ as illustrated in figure 1.

The following system of equations are incorporated for mathematical model [4].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+v\nabla^2 u \tag{2}$$

$$\frac{\partial v}{\partial x}u+\frac{\partial v}{\partial y}v=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+v\nabla^2 v \tag{3}$$

In equation 2. u and v are horizontal and vertical components of velocities, p the fluid pressure, base fluid density is ρ_f, v denotes kinematic viscosity.

$$\frac{\partial T}{\partial x}u+\frac{\partial T}{\partial y}v=\alpha\nabla^2 T+\tau\left[D_B(\nabla\phi.\nabla T)+\left(\frac{D_T}{T_\infty}\right)\nabla T.\nabla T\right]+\frac{v}{\rho_f}\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

$$\frac{\partial \varphi}{\partial x}u+\frac{\partial \varphi}{\partial y}v=D_B(\nabla^2\varphi)+\frac{D_T}{T_\infty}(\nabla^2 T) \tag{5}$$

In above equations, fluid temperature is T and T_∞ is the ambient temperature. C is nanoparticles concentration, C_∞ exhibits the free stream concentration. Brownian diffusion coefficient D_B, D_T denotes the thermophoretic diffusion coefficient.

The boundary conditions are:

$$\left.\begin{array}{l} u=U_w(x), v=0, T=T_w, \varphi=\varphi_w, \text{at } y=0 \\ u=v=0, T=T_\infty, \varphi=\varphi_\infty, \text{as } y\to\infty \end{array}\right\} \tag{6}$$

Now, define ψ where ψ be a stream function satisfying the continuity equation.

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$$

Now introduce the following similarity transformations:

$$\eta=y\sqrt{\frac{a}{v}},\quad \psi=\sqrt{va}x\,f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \beta(\eta)=\frac{\varphi-\varphi_\infty}{\varphi_w-\varphi_\infty} \tag{7}$$

The governing equations (2) – (5) are reduced into the following nonlinear ODEs:

$$f'''+ff''-\left(f'\right)^2=0 \tag{8}$$

$$\frac{\theta''}{Pr}+f\theta'+Nb\theta'\beta'+Nt(\theta')^2+Ec(f'')^2=0 \tag{9}$$

$$\beta''+Lef\beta'+\frac{Nt}{Nb}\theta''=0 \tag{10}$$

The associated boundary conditions (6) get the form:

At $\eta=0$,

$$f(\eta)=0, f'(\eta)=1, \theta(\eta)=1, \beta(\eta)=1 \tag{11}$$

At $\eta\to\infty$,

$$f'(\infty)\to 0, \theta(\infty)\to 0, \beta(\infty)\to 0 \tag{12}$$

Different parameters applied in the above equations have the following formulations:

$Pr=\frac{v}{\alpha}$, is Prandtl number,

$Le=\frac{a}{D_B}$ is Lewis number,

$Nb=\frac{\rho_p D_B(C_w-C_\infty)}{(\rho c)_f}$ is Browian motation parameter,

$Nt=\frac{\rho_p D_T(T_w-T_\infty)}{(\rho c)_f vT_\infty}$ is thermophoresis parameter and $Ec=\frac{u^2}{(\rho c)_f(T_w-T_\infty)}$ is the viscous dissipation parameter.

The skin friction, the Nusselt numbers and Sherwood number are characterized as:

$$C_f=\frac{\tau_w}{\rho u_w^2}, Nu_x=\frac{xq_w}{k((T_w-T_\infty)}, Sh_x=\frac{xh_m}{D_B((\varphi_w-\varphi_\infty)} \tag{13}$$

Here, τ_w , q_w and h_w are given as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}, h_m=-D_B\left(\frac{\partial\varphi}{\partial y}\right)_{y=0} \tag{14}$$

Using Eqns. (13) and (14):

$$C_f\sqrt{R_x}=f''(0), \frac{Nu_x}{\sqrt{R_x}}=-\theta'(0), \frac{Sh_x}{\sqrt{R_x}}=-\beta'(0) \tag{15}$$

here $R_x=ax^2$ is the local Reynolds number.

3. Computational Method

Equation (8) with the relevant boundary conditions Eqns. (11) and (12) has the exact solution is given by:

$$f(\eta)=1-e^{-\eta} \tag{16}$$

Knowing $f(\eta)$, we solved the equations (9) and (10) with the initial conditions

$$\theta(0)=1, \theta'(0)=\alpha_1, \beta(0)=1, \beta'(0)=\alpha_2 \tag{17}$$

(assumed α_1 and α_2)

Correct values of α_1 and α_2 are iteratively found using Newton's method by finding $\Theta=\frac{\partial\theta}{\partial\alpha_1}$, $\Phi=\frac{\partial\beta}{\partial\alpha_1}$, $\Theta=\frac{\partial\theta}{\partial\alpha_2}$, $\Phi=\frac{\partial\beta}{\partial\alpha_2}$ which are obtained by solving the equations at η_{max}.

$$\Theta''+Pr\left[f\Theta+Nb\left(\Theta'\phi'+\Phi'\theta'\right)+2Nt\,\theta'\Theta'\right]=0 \tag{18}$$

$$\Phi''+\left(\frac{Nt}{Nb}\right)\Phi'+Lef\Theta=0 \tag{19}$$

with initial conditions

$$\Theta(0)=0, \Phi(0)=1, \Theta'(0)=0, \Phi'(0)=0$$

once and with

$$\Theta(0)=0, \Phi(0)=0, \Theta'(0)=0, \Phi'(0)=1$$

Finally, in the entire computation every initial value problem is solved using by 4th order Adams-Bashforth Moulton method.

4. Code Validation

The given Table 4.1 and Table 4.2 shows the code validation of computed values with [4,18,19] and strong agreement with the values is found which the physical parameters, C_f , Nu_x and Sh_x The skin-friction coefficient examines the viscous stress acting on the surface of the body whereas Nu_x is the ratio between the convective to the conductive heat transfer to the boundary. Table 4.3 peruades the numerical values of Nu_x and Sh_x as a result of variation in the above parameters.

Table 4.1 Comparison of present results for Nu_x.

Pr	Present results $Nt = Nb = 0$	[4]	[18]	[19]
7	1.8956	1.8954	1.8954	1.8905
20	3.35391	3.3539	3.3539	3.3539
70	6.46228	6.4621	6.4622	6.4622

Table 4.2 Values of $-\theta'(0)$and $-\beta'(0)$ for $Pr = Le = 10, Ec = 0$.

Nt	Nb	Reduced Nusselt Number		Reduced Sherwood Number	
		W.A.Khan, I.Pop[4]	Present Result	W.A.Khan, I.Pop[4]	Present Result
0.1	0.1	0.9524	0.9523311	2.1294	2.129654
0.2	0.1	0.6932	0.6930403	2.274	2.275001
0.3	0.1	0.5201	0.519794	2.5286	2.531357
0.4	0.1	0.4026	0.402026	2.7952	2.801879
0.5	0.1	0.3211	0.3200067	3.0351	3.050751
0.1	0.2	0.5056	0.5055407	2.3819	2.382044
0.1	0.3	0.2522	0.2521248	2.41	2.410168
0.1	0.4	0.1194	0.1193849	2.3997	2.399787
0.1	0.5	0.0543	0.0542402	2.3836	2.383702

Table 4.3 Numerical results of $-\theta'(0)$and $-\beta'(0)$ with Ec, Nb and Nt for $Pr = Le = 10$

Ec	Nt	$Nb = 0.1$		$Nb = 0.2$		$Nb = 0.3$		$Nb = 0.4$		$Nb = 0.5$	
		Nur	Shr	Nur	Shr	Nur	Shr	Nur	Shr	Nur	Shr
0	0.1	0.952331	2.129654	0.505541	2.382044	0.252124	2.410167	0.119383	2.399787	0.054239	2.383702
0	0.2	0.693041	2.275001	0.365248	2.515838	0.181518	2.515498	0.08585	2.481194	0.03901	2.447238
0	0.3	0.519794	2.531358	0.272869	2.657117	0.135355	2.61015	0.063981	2.549797	0.029077	2.499496
0	0.4	0.402026	2.801879	0.210551	2.785812	0.104309	2.690817	0.049278	2.6067	0.022393	2.542549
0	0.5	0.320007	3.050751	0.167269	2.897635	0.082746	2.759224	0.039049	2.654762	0.017729	2.579189
0.5	0.1	0.0457149	2.861791	-0.2644993	2.657423	-0.3573054	2.54849	-0.3701797	2.479211	-0.3476905	2.433646
0.5	0.2	-0.2071292	3.598767	-0.3607857	3.026319	-0.4035311	2.776947	-0.3901633	2.633362	-0.3550929	2.54375
0.5	0.3	-0.3418508	4.403095	-0.4320656	3.389679	-0.4381708	2.988962	-0.4054824	2.771534	-0.3610229	2.640588
0.5	0.4	-0.4465305	5.230272	-0.4884278	3.742773	-0.4662342	3.18716	-0.4183187	2.897538	-0.3662797	2.727711
0.5	0.5	-0.5333369	6.081104	-0.5362393	4.093562	-0.4905265	3.378976	-0.4296258	3.017568	-0.3709825	2.810239
2.0	0.1	-3.522905	5.520873	-2.954666	3.629356	-2.422955	3.019192	-1.981306	2.739736	-1.638313	2.592446
2.0	0.2	-4.27376	9.77462	-3.460486	5.257116	-2.728654	3.825445	-2.153128	3.192596	-1.731986	2.873283
2.0	0.3	-6.108167	17.83333	-4.694522	8.082056	-3.419755	5.022495	-2.499911	3.759993	-1.901296	3.180815
2.0	0.4	-12.84432	46.93884	-9.933496	19.04211	-6.114932	8.834164	-3.455805	4.920121	-2.24068	3.624038

5. Results and Discussion

The demonstration of this part is to analyse numerical results represented in the form of graphs. The calculation has been made for different estimations of Brownian motion, Eckert Number, Thermophoresis, Lewis number and Prandtl number.

5.1 Effect of Prandtl number ($\boldsymbol{Pr}$)

Figure 2 is presented in order to analyse the effect of Prandtl number on temperature profile. The temperature in the boundary layer decreases as a result of the increasing Pr and the thickness of boundary layer also decreases. Figure 3 depicted the influence of Pr on concentration $\beta(\eta)$. $\beta(\eta)$increases with the increase of Pr. The increase of $\beta(\eta)$ due to increase of Pr is far away from surface. Heat source is higher than heat sink in temperature for fixed Prandtl number.

5.2 Effect of Lewis number ($\boldsymbol{Le}$)

The impact of Le on dimensionless temperature profile $\theta(\eta)$ can be observed in Figure 4. From the figure, it is detected that by increasing values of Lewis number temperature near the surface of plate decreases and away from the surface of plateboosts. Physically Le is the ratio of thermal diffusion to the rate of mass diffusion. Figure 5. reflects the impact of Le on dimensionless concentration profile. Le can be defined as the ratio of thermal diffusion to the molecular diffusion. It is convenient of help us find the relation between mass and heat transfer coefficient. By increasing Lewis number, the concentration profile becomes steeper.

5.3 Effect of Thermophoresis ($\boldsymbol{Nt}$)

The impact of thermophoresis parameter on the dimensionless temperature profile $\theta(\eta)$ and dimensionless profile of concentration distribution $\beta(\eta)$ are presented respectively in Figures 6 and 7. It is clear, from these figures profile of temperature and their associative thermal boundary layer thickness of the thermal field increase with the increasing values of thermophoresis parameter. It is also perceived that for varying values of Nt concentration profile $\beta(\eta)$ and related thickness of boundary layer increases.

5.4 Effect of Brownian motion ($\boldsymbol{Nb}$)

Figures 8 exhibits the impact of Brownian motion in temperature and Figure 9 indicates the effect of Nb on concentration. The temperature increases with the boost of Brownian motion whereas concentration profile decreases significantly.

5.5 Effect of Eckert number ($\boldsymbol{Ec}$)

Figures 10 and 11 displays the influence of Eckert number on the energy and mass transfer profiles. It can be

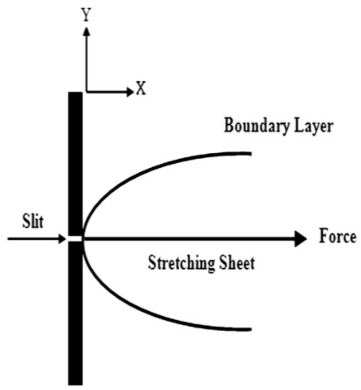

Figure 1. Flow Geometry

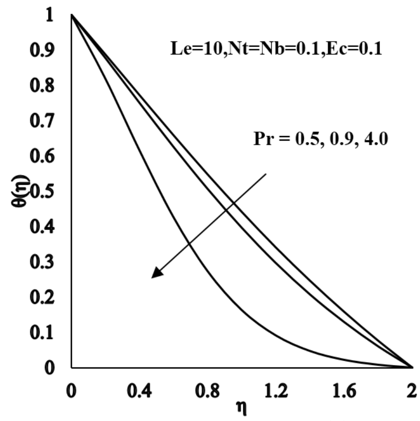

Figure 2. Effect on Pr on $\theta(\eta)$

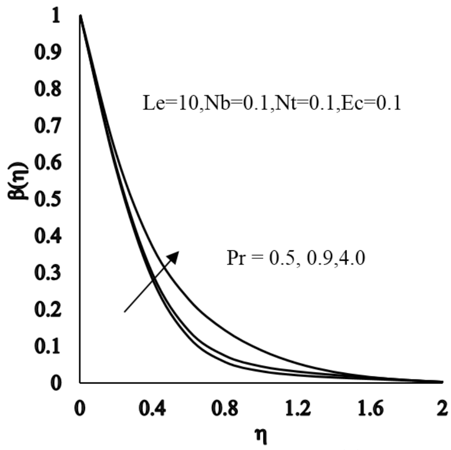

Figure 3. Impact of Pr on $\beta(\eta)$

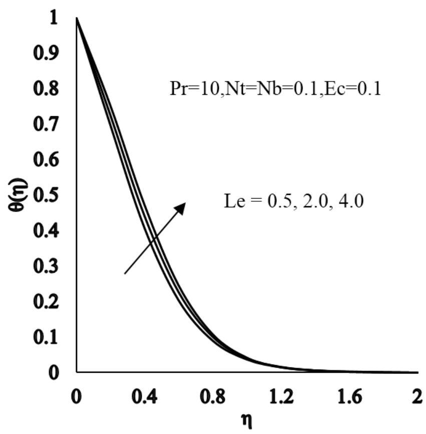

Figure 4. Influence of Le on $\theta(\eta)$.

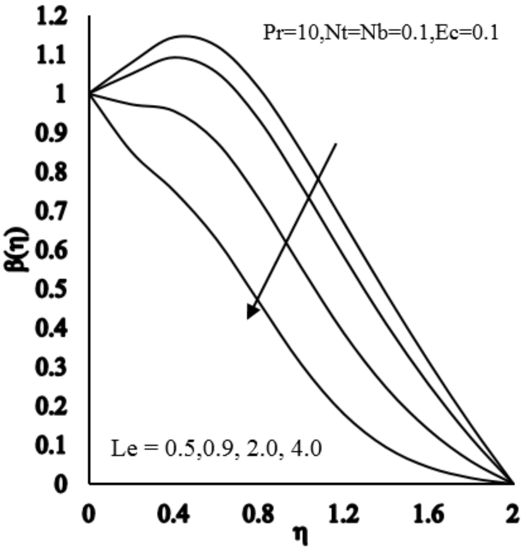

Figure 5. Impact of $e\ \beta(\eta)$

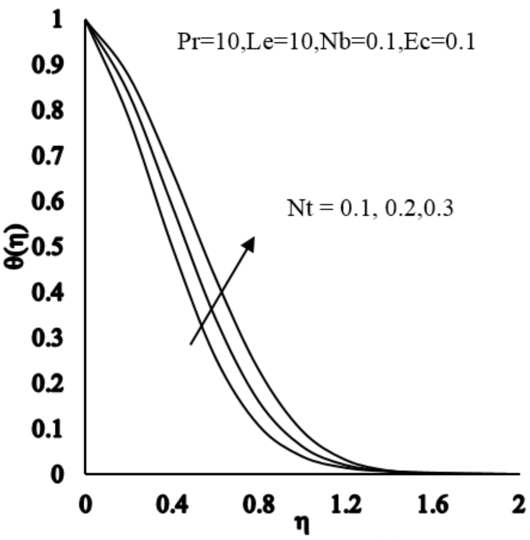

Figure 6. Impact of Nt on $\theta(\eta)$

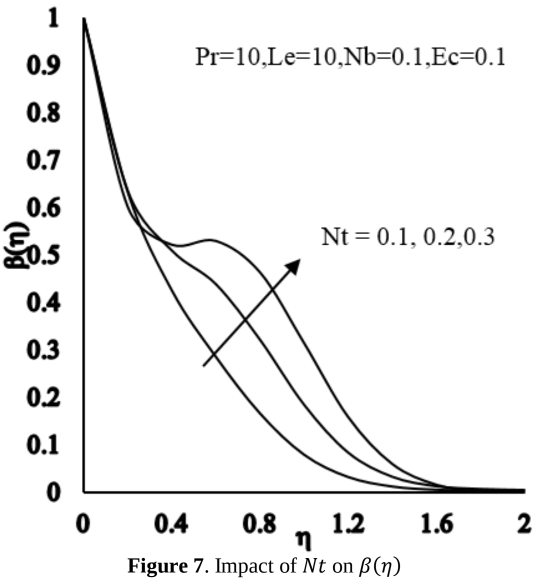

Figure 7. Impact of Nt on $\beta(\eta)$

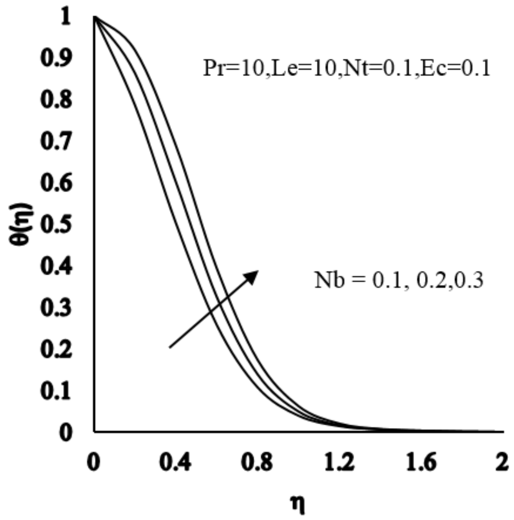

Figure 8. Effect of Nb on $\theta(\eta)$.

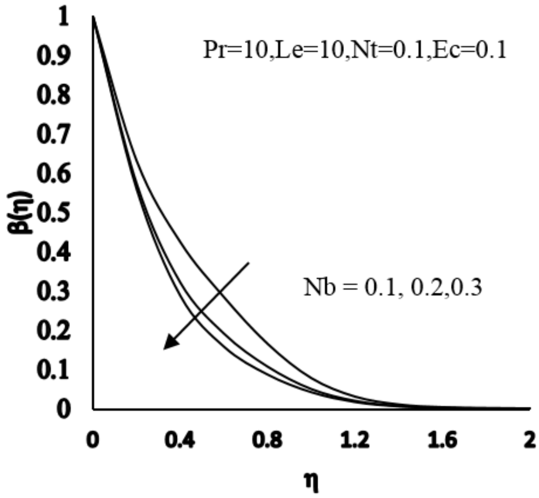

Figure 9. Effect of Nb on $\beta(\eta)$.

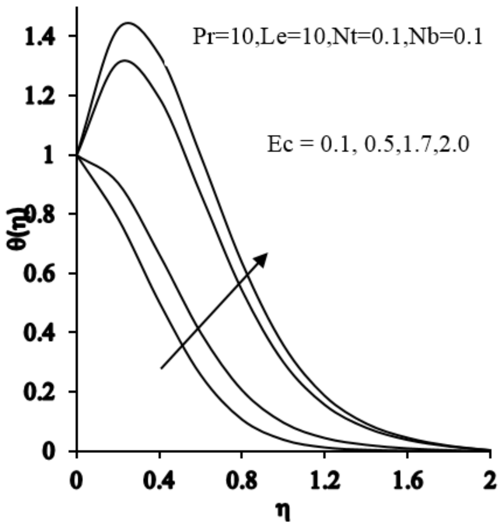

Figure 10. Impact of Ec on $\theta(\eta)$.

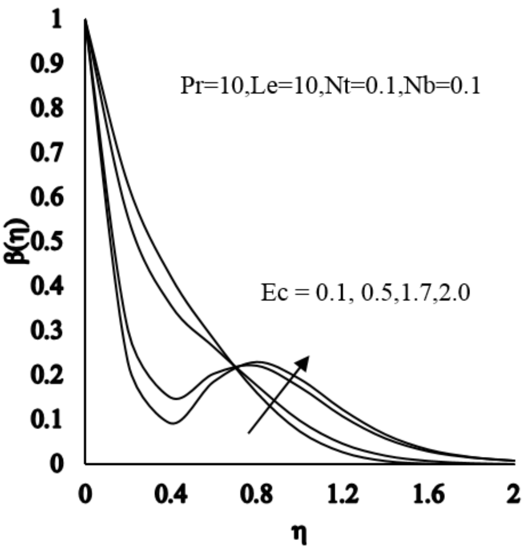

Figure 11. Impact of Ec on $\beta(\eta)$.

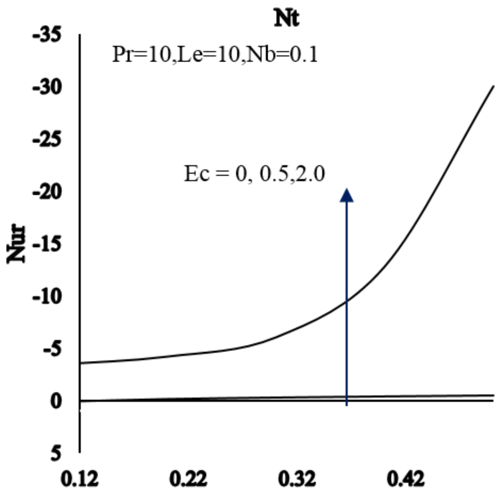

Figure 12. Impact of Ec on $-\theta^{'}(\eta)$

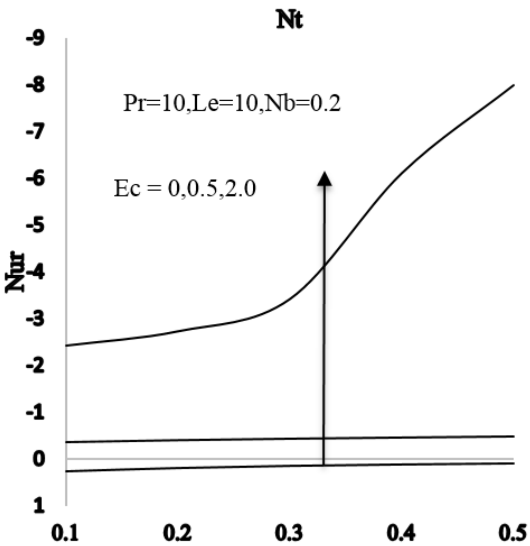

Figure 13. Behaviour of Ec on $-\theta^{'}(\eta)$

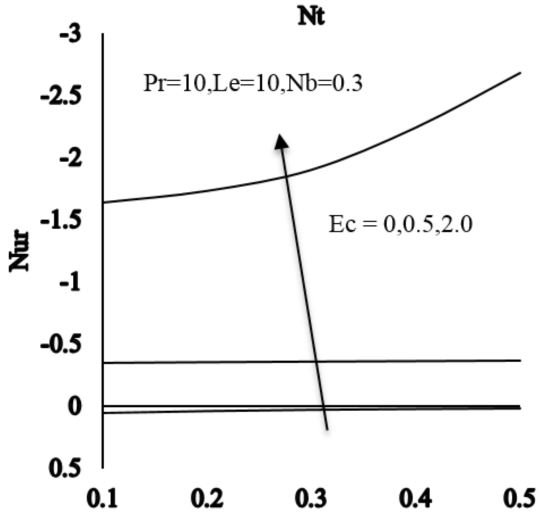

Figure 14. Behaviour of Ec on $-\theta^{'}(\eta)$

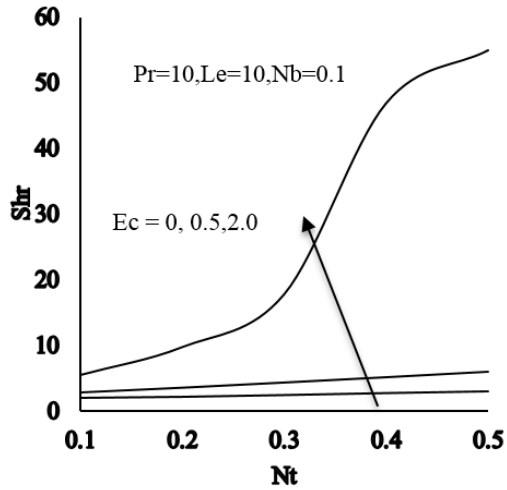

Figure 15. Behaviour of Ec on $-\beta^{'}(\eta)$

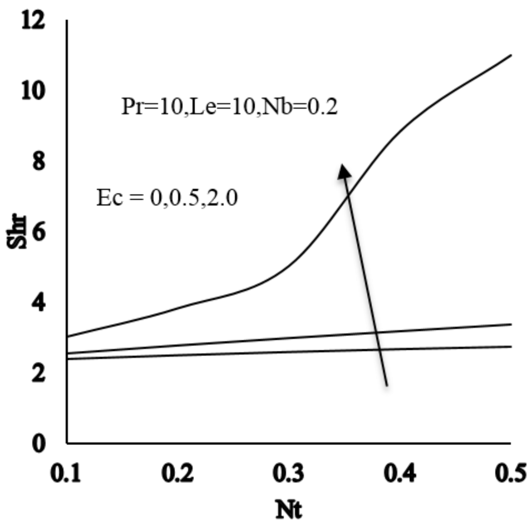

Figure 16. Behaviour of Ec on $-\beta^{'}(\eta)$

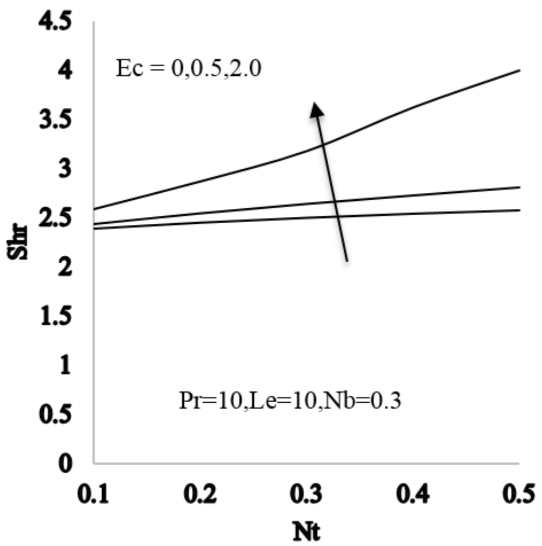

Figure 17. Behaviour of Ec on $-\beta^{'}$ (η)

observed that energy profile and mass transfer profiles are increased when the $\boldsymbol{Ec}$ isboosted. Due to friction, the heat energy is kept in owing to accelerating values of Eckert number, which results in the enhancement of the temperature profile.

Variation in $-\boldsymbol{\theta}^{'}$ $(\boldsymbol{\eta})$ and $-\boldsymbol{\beta}^{'}$ $(\boldsymbol{\eta})$ against Thermophoresis parameter $(\boldsymbol{Nt})$ is shown in Figures. 12 to 17 for fixed values of Pr and Le and three values of Nb.

6. Conclusion

From the above discussion, we can make the following conclusions.

- Increase in viscous dissipation increases temperature and concentration profile.
- By increasing the thermophoresis parameter Nt increases concentration profile.
- An increase in Brownian motion parameter increases temperature, while concentration decreases in the horizontal direction.
- On temperature profile, Prandtl number Pr has decreasing effects. Whereas a rise in concentration profile.
- For larger values of Lewis number, concentration field shows decreasing behavior.

Nomenclature

a	Constant (s^{-1})
C_f	Skin friction coefficient
C_w	Nanoparticles volume fraction at the stretching sheet
C_∞	Ambient nanoparticles volume
D_B	Brownian diffusion coefficient
D_T	Thermophoresis diffusion coefficient
E_c	Eckert number
$f(\eta)$	Dimensionless stream function
κ	Thermal conductivity$(Wm^{-1}K^{-1})$.
Le	Lewis number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Nu	Nusselt number
Nur	Reduced Nusselt number
Pr	Prandtl number
p	Fluid pressure
$(\rho c)_f$	Heat capacity of the fluid $(Jm^{-3}K^{-1})$
$(\rho c)_p$	Effective heat capacity of the nanoparticle material $(Jm^{-3}K^{-1})$
q_m	Wall mass flux
q_w	Wall heat flux
Re_x	Local Reynolds number
Shr	Reduced Sherwood number
Sh_x	Local Sherwood number
T	Fluid temperature (K)
T_w	Temperature at the stretching sheet (K)
T_∞	Ambient temperature (K)
u_w	Velocity of the stretching sheet (ms^{-1})
u, v	Cartesian coordinates (x axis is aligned along the stretching surface and y axis is the normal to it) $(m.s^{-1})$
α	Thermal diffusivity (m^2s^{-1})
β	Dimensionless nanoparticles volume fraction
η	Similarity variable
θ	Dimensionless temperature
ν	Kinematic viscosity of the fluid
ρ_f	Fluid density (kgm^{-1})
ρ_p	Nanoparticle mass density (kgm^{-1})
σ	Electrical conductivity of the fluid
τ	Parameter defined by ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid. $\tau = (\rho c)_p/(\rho c)_f$.
φ	Nanoparticle volume fraction
φ_∞	Ambient nanoparticle volume fraction
φ_w	Nanoparticle volume fraction at the stretching sheet
ψ	Stream function (m^2s^{-1})
∞	Condition at the free stream
w	Condition of the surface

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