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Analysis of radiation heat transfer of a micropolar fluid with variable properties over a stretching sheet in the presence of magnetic field

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ABSTRACT

The present study deals with the analysis of the effects of radiative heat transfer of micropolar fluid flow over a porous and stretching sheet in the presence of magnetic field. The dynamic viscosity and thermal conductivity coefficient have been formulated by temperature-dependent relations to obtain more exact results. The flow is supposed two-dimensional, incompressible, steady and laminar and the applied magnetic field is assumed uniform. On the other hand, the velocity of the isothermal stretching sheet varies linearly with the distance from a fixed point on the sheet. The governing equations have been extracted using the theory of micropolar fluid and the boundary layer approximation. Then they have been solved by similarity solution relationships, shooting method and fourth-order Runge-Kutta method. The results express that the presence and increase of variable thermal conductivity parameter, magnetism, radiation and variable viscosity parameter cause to decrease heat transfer from the sheet, while increase of material parameter, Prandtl number and suction parameter increases the rate of heat transfer from the sheet. Also, the values of dimensionless velocity are enhanced by an increase in variable thermal conductivity parameter, material parameter and radiation parameter. On the other hand, the values of dimensionless angular velocity are completely influenced by the values of the velocity gradient.

Introduction

Developing functional and industrial processes in manufacturing the materials, extrusion process, continuous glass casting, heating the sheet like materials, cooling the filaments and strips in polymer industries and so forth, reveals the important role of fluid flow on the sheets. Therefore, in the last few decades, considerable researches have been done to study the characteristics of the flow over a sheet and heat transfer rate from the surface. On the other hand, The presence of the thermal radiation and magnetic field is a significant factor in the growth of the boundary layers, heat transfer rate and the final quality of the products. Also, classical fluid dynamics could not study the behaviors of a lot of fluids such as micropolar fluids like a Newtonian one. Therefore, the presented model by Eringen [1], known as the theory of micropolar fluids, is used to consider this special kind of fluids, such as the flows of liquid crystals, colloidal fluids, bubbly liquids and ferrofluids. This mathematical model extracts the equations of flow and energy to investigate the innate polar property by conception of micro rotation.

Some valuable researches [2-6] describe the micropolar fluid flow over a stretching sheet and find out that unlike the blowing, the effects of suction

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increase the rate of heat transfer from the sheet while the magnitudes of fluid velocity decrease; besides, the enhancement of thermal conductive fluid and dynamic viscosity declines the cooling rate of the surface. Other studies dealt with the analysis of micropolar fluid flow on a shrinking sheet [7-9] to determine the range of suction parameter for existing dual solutions in different values of other parameters. Also, some studies [10-19] consider the influences of thermal radiation and magnetic field on micropolar fluid flow, and explain that the increase of radiative heat flux and Joule heating resulting from magnetic field raise the temperature velocity values in the boundary layers. On the other hand, several investigations [20-22] concentrate on microstructure effects of micropolar fluid flow, and express he influences of micro rotation and micro motion features on dimensionless profiles of velocity, angular velocity and temperature. In addition, the application of micropolar fluid in industrial process, such as extruding a polymeric sheet [23], and the widespread applications of modeling the fluids such as micropolar ones in medical fields [24] show the importance of such fluids in practical issues.

This study can provide the integral background for improvement of thermal efficiency of the porous insulation, applied in high-temperature furnaces. The importance of considering magnetic field and thermal radiation, resulting from such a high temperature in many complex industrial uses, and the needs for more efficient porous insulation necessitate studying new fluids and methods to improve the thermal function of insulation, which is the use of micropolar fluid motion due to a stretching sheet, in the present study.

Here, the flow is supposed to be laminar, two-dimensional, steady and incompressible, and the body forces are neglected. The applied magnetic field is considered uniform along the sheet and the thermal condition of the sheet is supposed to be isothermal. Furthermore, the sheet is stretched linearly and it is assumed to be permeable. What distinguishes the problem in this research from other studies is the simultaneous presence of thermal radiation, magnetic field and micropolar fluid flow with variable viscosity and thermal conductivity over a permeable stretching sheet. The results include the profiles of velocity, angular velocity and temperature, and discussion about the influences of contributing factors in the variations of the profiles.

1. Governing Equations

As it is shown in figure 1, moving a micropolar fluid flow over a stretching sheet leads to the boundary layer formation. The x-axis is chosen along the sheet, and the y-axis is taken normal to it. Using the theory of micropolar fluid and boundary layer approximation leads to the following governing equations [25]:

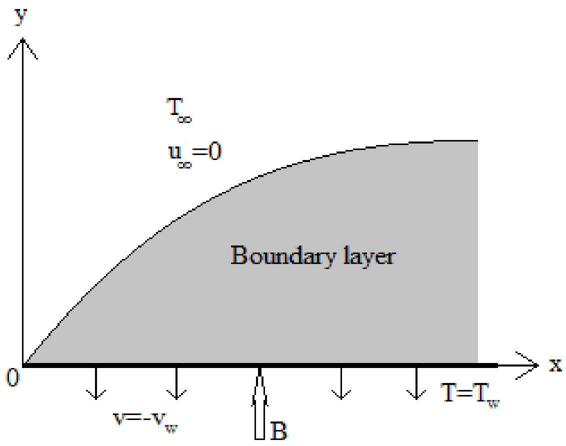

Fig. 1 The physical schematic of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}=\frac{1}{\rho}\frac{\partial}{\partial y}\left((\mu+S)\frac{\partial u}{\partial y}\right)+\frac{S}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma u B^2}{\rho} \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\left(\mu+\frac{S}{2}\right)\frac{\partial N}{\partial y}\right)-\frac{S}{\rho j}(2N+\frac{\partial u}{\partial y}) \tag{3}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{\partial q_r}{\partial y}+\sigma B^2u^2 \tag{4}$$

Equations (1-4) describe the conservation of mass, momentum, angular momentum and energy for this problem, respectively. Because the micro inertia is considered constant, the equation of micro inertia conservation is not applied. In Eq. (4), q_r is the radiative heat flux, which is approximated using Rosseland equation [26] using assuming the large optical thickness to get a diffusion medium.

$$q_r=-(\frac{4\xi}{3\kappa})\frac{\partial T^4}{\partial y} \tag{5}$$

By replacing the expansion of T^4 in Taylor series as follows:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots\approx-3T_\infty^4+4T_\infty^3T \tag{6}$$

The energy equation is transformed into the following:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{16\xi T_\infty^3}{3\kappa}\frac{\partial^2 T}{\partial y^2}+\sigma B^2u^2 \tag{7}$$

According to the assumed physical conditions, the boundary conditions at the surface (y=0) are written by:

$$u = cx, c > 0 \quad \text{(8-a)}$$
$$v = -v_w, v_w > 0 \quad \text{(8-b)}$$
$$N = -m\left.\frac{\partial u}{\partial y}\right|_{y=0} \quad \text{(8-c)}$$
$$T = T_w \quad \text{(8-d)}$$

The Equations (8-a, 8-b) state the linear velocity of the stretching sheet and the constant velocity of fluid penetration into the porous sheet, respectively. By using the theory of micropolar fluid flow, the angular velocity is defined [27] as Eq. (8-c) in which the constant m, as micro rotation parameter can be a value from 0 to 1. m=0 represents the concentrated particle flow and the microelements are close to the wall and cannot be rotated [28], this status is known by strong focus on the microelements [29]. Also, m=0.5 states the vanishing asymmetric part of stress tensor and it is known by weak focus on the microelements [30]. On the other hand, the turbulent boundary layer of a micropolar fluid could be modeled by m=1 [31].

The total spin of a micropolar fluid flow is caused by micro rotation and macro rotation effects of the flow, which are dominant near the wall and far from the wall, respectively. If the focus on microelements is weak (m≠0), microstructure effects of micropolar fluids are neglected and the rotation exists just because of the shear stress [30]. In this study, the governing equations are solved and the results are extracted for m=0.5, so, the micro rotation is equivalent to angular velocity in Newtonian fluids, like the behaviors of red cells in the vessels [32].

The boundary conditions in the areas far from the wall (y→∞) are as follows:

$$u \to 0 \quad \text{(9-a)}$$
$$N \to 0 \quad \text{(9-b)}$$
$$T \to T_\infty \quad \text{(9-c)}$$

In this study, the applied temperature-dependent relationship for thermal conductivity coefficient is considered linear [33] which is applicable for an extensive range of temperature and is written as follows [34]:

$$k = k_\infty\left(1 + \varepsilon\frac{T - T_\infty}{T_w - T_\infty}\right) \quad (10)$$

In Eq. (10), ε is the variable thermal conductivity parameter which is defined by the following [34]:

$$\varepsilon = \frac{k_w - k_\infty}{k_\infty} \quad (11)$$

Despite of several presented temperature-dependent relationships for defining the dynamic viscosity, such as Reynolds viscosity model [35] or Vogel's viscosity model [36], the following relationship which is more suitable for a wide range of temperatures, is used [37, 38]:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] \quad (12)$$

In Eq. (12), γ is the thermal property of the fluid, which is a positive value for liquids and negative value for gases, provided that the wall temperature is more than the fluid's [39].

Since the governing equations are parabolic, and the flow is supposed to be two-dimensional and incompressible, similarity solution could be used and stream function would be applied to create similarity relations for transformation of Partial Differential Equations (PDEs) into Ordinary Differential Equations (ODEs). Therefore, the following non-dimensional variables are introduced:

$$\begin{aligned} &u = \partial\psi/\partial y \\ &v = -\partial\psi/\partial x \\ &\eta = (c/v_\infty)^{1/2} y \\ &\psi = (cv_\infty)^{\frac{1}{2}} x f(\eta) \\ &N = cx(c/v_\infty)^{\frac{1}{2}} h(\eta) \\ &T = T_\infty + (T_w - T_\infty)\theta(\eta) \end{aligned} \quad (13)$$

By substituting the above similarity transformation, Eq. (10) and Eq. (12) are transformed into the following:

$$k = k_\infty(1 + \varepsilon\theta(\eta)) \quad (14)$$
$$\mu = \mu_\infty\left(\frac{\theta_r}{\theta_r - \theta(\eta)}\right) \quad (15)$$

In Eq. (14), θ_r is the variable viscosity parameter, which is defined by:

$$\theta_r = -\frac{1}{\gamma(T_w - T_\infty)} \quad (16)$$

By substituting Equations (13-15) the governing equations are transformed into non-linear ODEs, as follows:

$$\begin{aligned} &\left[\frac{\theta_r}{\theta_r - \theta(\eta)} + \Delta\right] f'''(\eta) \\ &+ \frac{\theta_r}{\left(\theta_r - \theta(\eta)\right)^2} f''(\eta)\theta'(\eta) \\ &+ f(\eta)f''(\eta) - f'^2(\eta) + \Delta h'(\eta) \\ &- (Ha)^2 f'(\eta) = 0 \end{aligned} \quad (17)$$

$$\left[\frac{\theta_r}{\theta_r-\theta(\eta)}+\frac{\Delta}{2}\right]h''(\eta) + \frac{\theta_r}{\left(\theta_r-\theta(\eta)\right)^2}h'(\eta)\theta'(\eta)+f(\eta)h'(\eta) - f'(\eta)h(\eta)-\Delta[2h(\eta)+f''(\eta)]=0 \quad (18)$$

$$[(1+\varepsilon\theta(\eta))+R]\theta''^{(\eta)}+\varepsilon\theta'^2(\eta) + Pr_\infty f(\eta)\theta'(\eta) + Pr_\infty (Ha)^2 Ecf'^2(\eta) = 0 \quad (19)$$

The boundary conditions are transformed into the following:

$$f(\eta)=M \quad (20\text{-a})$$
$$f'(\eta)=1 \quad \text{at } \eta=0 \quad (20\text{-b})$$
$$h(\eta)=-mf''(\eta) \quad (20\text{-c})$$
$$\theta(\eta)=1 \quad (20\text{-d})$$

And

$$f'(\eta)\to 0,\quad h(\eta)\to 0,\quad \theta(\eta)\to 0 \quad \text{at } \eta\to\infty \quad (21)$$

In the above equations, the dimensionless parameters are as follows:

$$Pr_\infty=\frac{\mu_\infty C_p}{k_\infty},\ Ec=\frac{c^2x^2}{C_p(T_w-T_\infty)},\ R=\frac{16\xi T_\infty^3}{3k_\infty\kappa},$$
$$Ha=B\sqrt{\frac{\sigma}{\rho c}},\ \Delta=\frac{S}{\mu_\infty},\ \mathrm{M}=\frac{v_w}{(c\nu_\infty)^{1/2}}$$

These dimensionless parameters are named as Prandtl number, Eckert number, radiation parameter, Hartmann number, material parameter and suction parameter, respectively from left to the right. Here, the shooting method is used to transform the boundary value problem into an initial value problem. In this method, the solution begins at the lower bound of the boundary value problem by shooting a guess, as the initial gradient at this bound; then the upper bound is compared with its correct value by fourth-order Runge-Kutta method, as an initial value problem solver, which is modified Taylor method, and works based on temporal discretization. This process is repeated until the boundary condition at the upper end converges to its right value. In the current study, such numerical trend is done in symbolic software Mathematica.

2. Results and Discussions

In this paper, the numerical results are obtained for each desired parameter, while the values of other dimensionless parameters are considered fixed to achieve the exact effects of the intended parameter on the fluid flow. All selective constant parameters are supposed unit except for variable viscosity parameter which is assumed minus two. To validate the results, the values of $-\theta'(0)$ in this research is compared with some references in Table 1, and because of the novelty of this work, they differ from this study in the features represented in the capture of Table 1. The close answers, and the very small amounts of deviation, as a measure of variability or volatility in the given set of data, for any given results certify the present results.

Table 1. The values of $-\theta'(0)$ for various values of R, Δ and Pr when Ha=0, M=0, m=0.5, ε=0 and $\theta_r\to\infty$

R	Δ	Pr	Grubka and Bobba [40]	Ali [41]	Chen [42]	Ishak [11]	Present	Deviation
0	0	0.72	0.4631	0.4617	0.46315	0.4631	0.463592	0.00071717
0	0	1	0.5820	0.5801	0.58199	0.5820	0.582011	0.00084985
0	0	3	1.1652	1.1599	1.16523	1.1652	1.16524	0.002378125
0	0	10	2.3080	2.2960	2.30796	2.3080	2.308	0.005362119
1	0	1				0.3547	0.35724	0.001796051
1	1	1				0.3893	0.391301	0.001414921

Figure 2 and figure 3 show the effect of variable thermal conductivity parameter on temperature profile and velocity profile, respectively. According to Eq. (14), an increase in variable thermal conductivity parameter causes to more thermal conductivity coefficient, which results in further thermal diffusion through the fluid. Hence, the values of dimensionless temperature in the thermal boundary layer increase and temperature gradient on the wall and heat transfer rate from the sheet dwindle. Based on Eq. (15), more values of temperature result in lower viscosity which accelerates the fluid motion.

Figure4 and figure 5 represent the effects of presence and increase of magnetic field intensity on the velocity and the temperature profiles, respectively. Applying magnetic field in an electrically conducting fluid creates a drag force called Lorentz force which reduces the fluid velocity. Consequently, an increase in Hartmann number reduces the rate of transport, and the thickness of the momentum boundary layer decreases. On the other hand, the resistance against the flow created by Lorentz force leads to more heat transport rate through the fluid known as Joule heating. Therefore, the values of temperature in the boundary layer increase and the absolute temperature gradients at the surface and heat transfer from the sheet decline.

Figure 6 and figure7 show the effects of the material parameter as an important characteristic of micropolar fluid, on velocity and temperature profiles, respectively. It is obvious that an increase in material parameter, thickness and the momentum boundary layer reduces the values of temperature profile, so the absolute values of temperature gradient are lowered and the heat transfer from the sheet is enhanced.

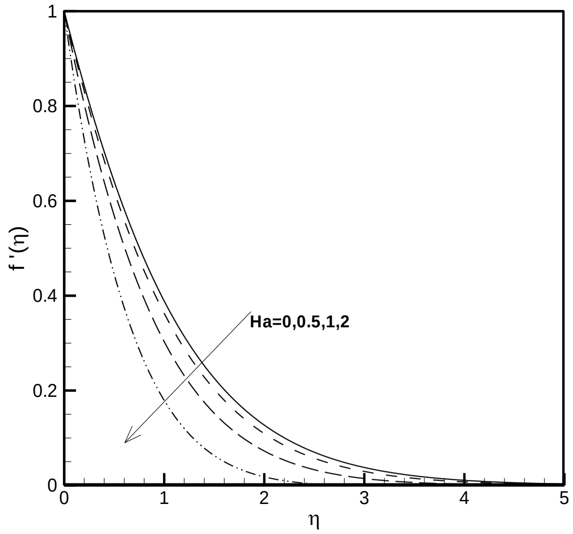

Fig. 4 The dimensionless velocity profiles for different values of Ha

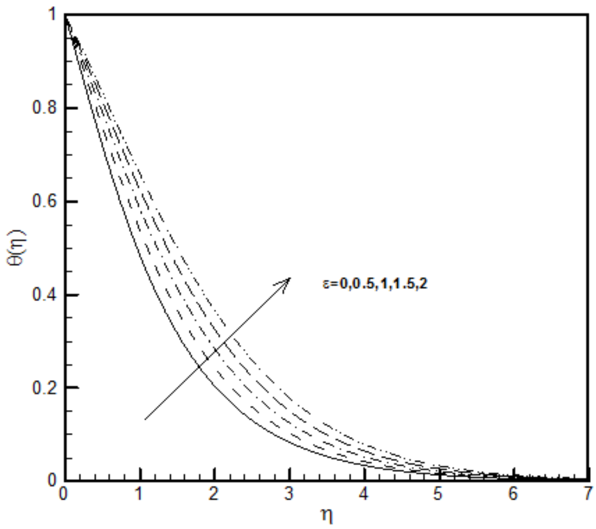

Fig. 2 The dimensionless temperature profiles for different values of ε

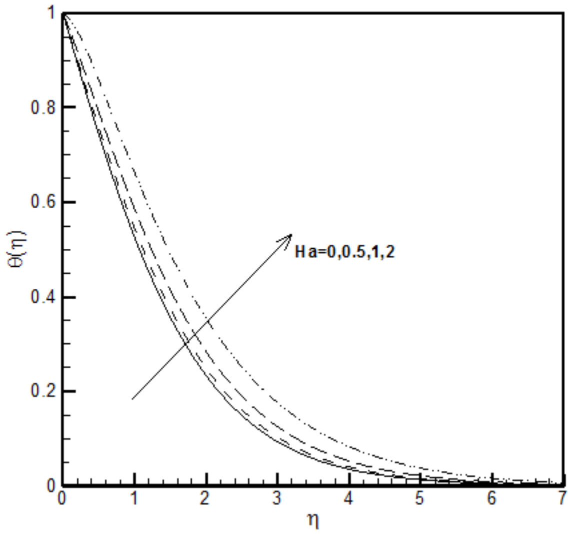

Fig. 5 The dimensionless temperature profiles for different values of Ha

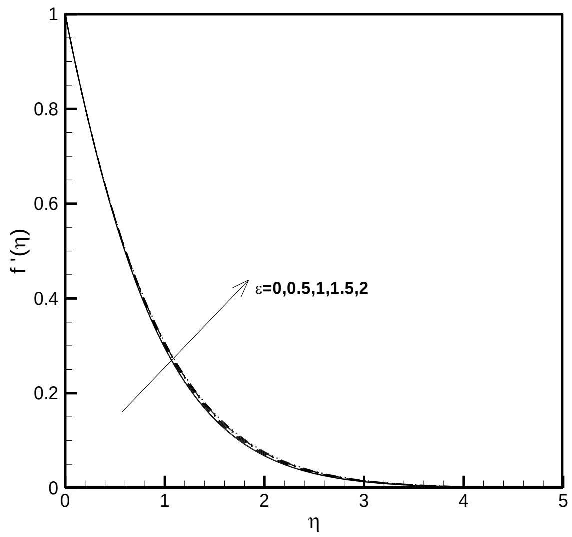

Fig. 3 The dimensionless velocity profiles for different values of ε

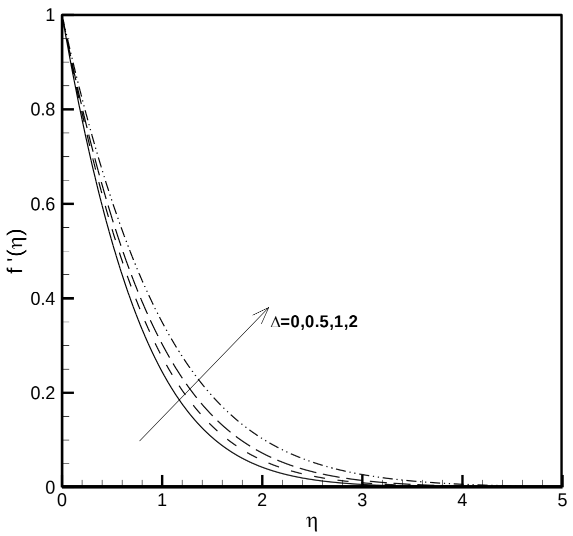

Fig. 6 The dimensionless velocity profiles for different values of Δ

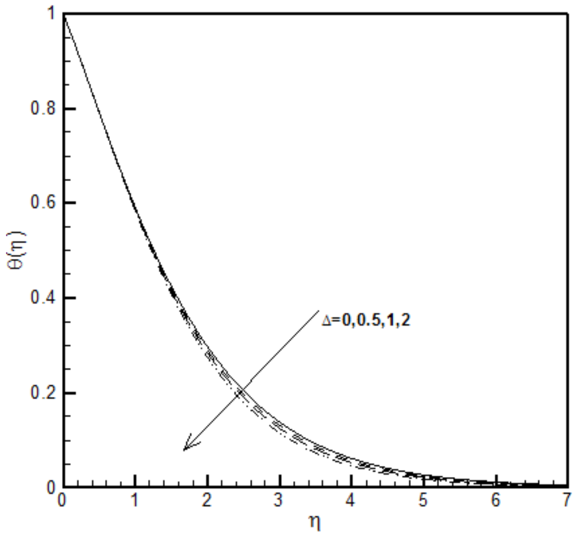

Fig. 7 The dimensionless temperature profiles for different values of Δ

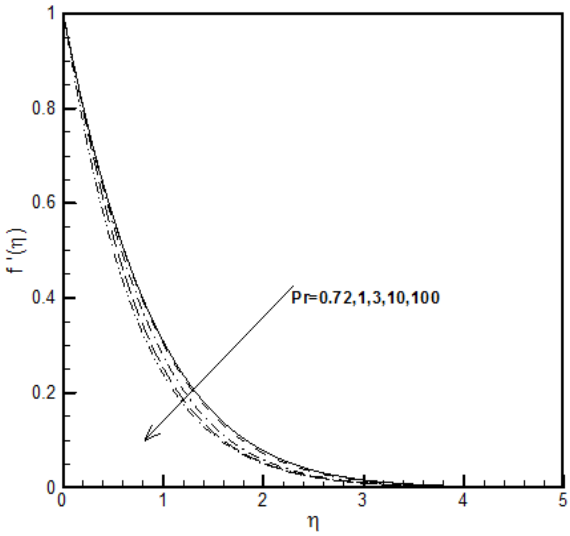

Fig. 8 The dimensionless velocity profiles for different values of Pr

Figure 8 and figure 9 depict the effects of Prandtl number on velocity and temperature profiles. Prandtl number is the ratio of viscous forces to thermal diffusion, so the addition of Pr number implies more resistant force in fluid motion and a reduction in velocity values. Also, less thermal diffusion leads to lower energy transport through the fluid and more absolute values of temperature gradient on the surface which leads to the enhancement of heat transfer rate from the sheet.

Figure 10 shows the effects of radiative heat transfer on temperature profiles by variations of thermal radiation parameter. The presence and increase of thermal radiation parameter augment the temperature values in the thermal boundary layer. According to the definition of radiation parameter, an increase in R implies decrease of absorption coefficient which leads to more radiative heat flux defined by Eq. (5), so the rate of energy transport to the fluid and the values of temperature distribution are raised. Finally, according to the Eq. (15), more temperature values result in lower viscosity and more velocity values (figure 11).

Figure12 displays the effect of suction parameter on velocity profiles. Augmentation of fluid penetration velocity into the sheet by enhancement of suction parameter leads to a reduction in total momentum in the flow direction, and then to lower dimensionless velocity values. Furthermore, figure 13 shows that more fluid penetration into the sheet provokes thinner thermal boundary layer and more heat transfer rate due to the faster replacement of heated fluid with the cooler fluid.

Figure14 and figure15 display the velocity profiles in various positive and minus values of variable viscosity parameters, respectively. An increase in θr, from theminimum negative value to the maximum positive value means growth of dynamic viscosity, leading into a reduction in thickness of the velocity boundary layer.

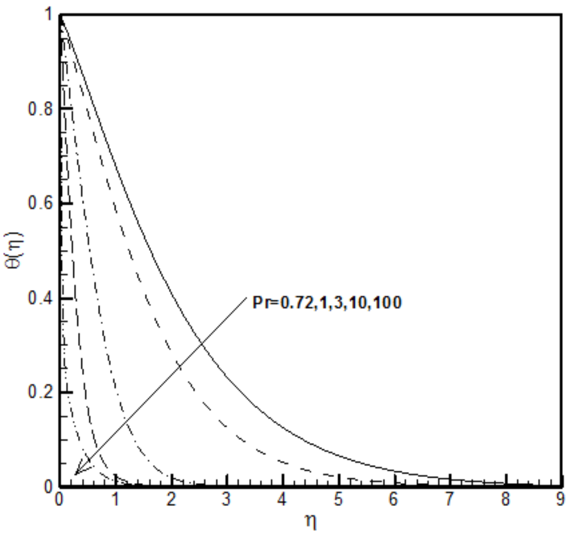

Fig. 9 The dimensionless temperature profiles for different values of Pr

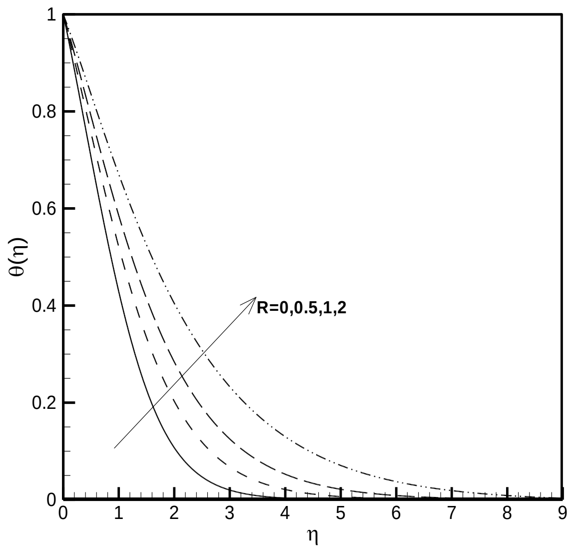

Fig. 10 The dimensionless temperature profiles for different values of R

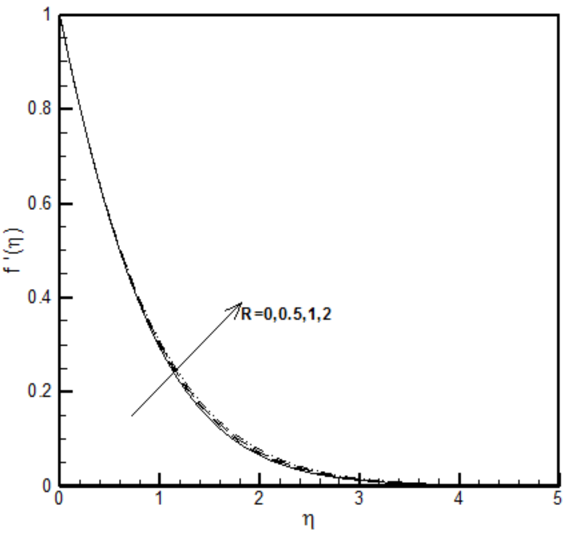

Fig. 11 The dimensionless velocity profiles for different values of R

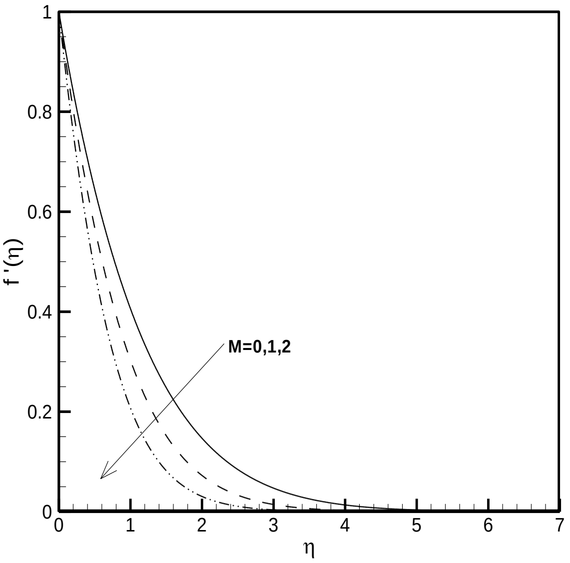

Fig. 12 The dimensionless velocity profiles for different values of M

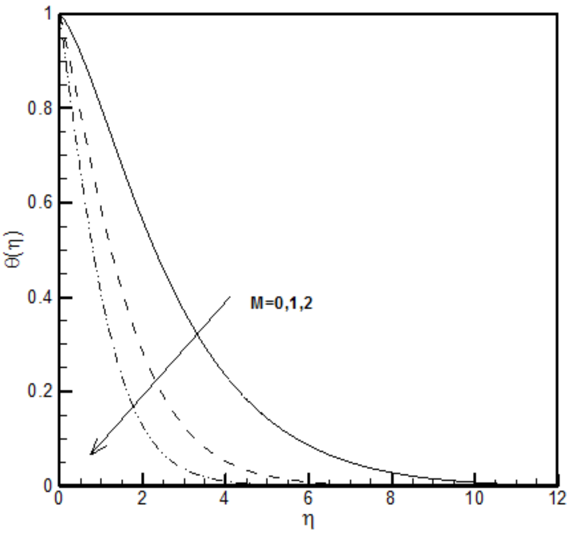

Fig. 13 The dimensionless temperature profiles for different values of M

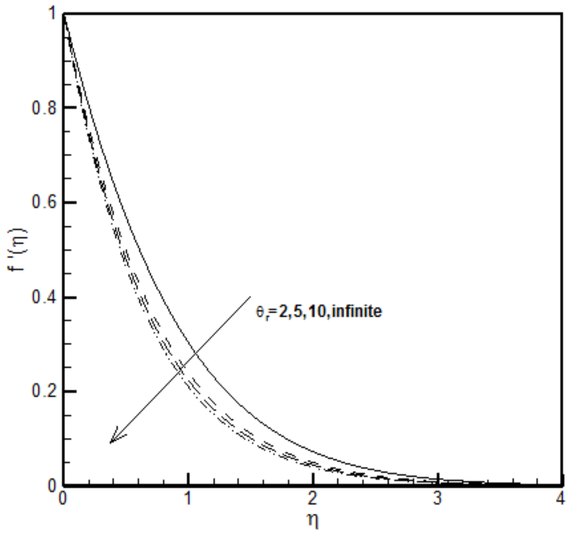

Fig. 14 The dimensionless velocity profiles for different positive values of θ_r

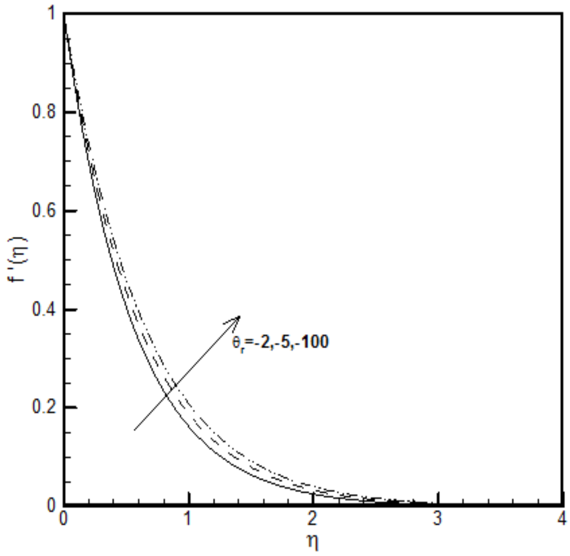

Fig. 15 The dimensionless velocity profiles for different minus values of θ_r

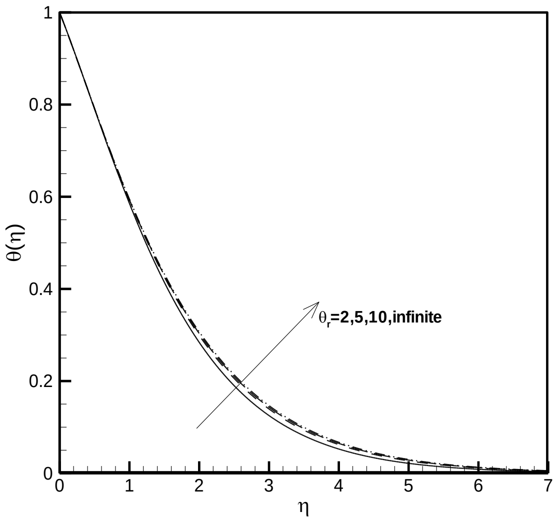

Fig. 16 The dimensionless temperature profiles for different positive values of θ_r

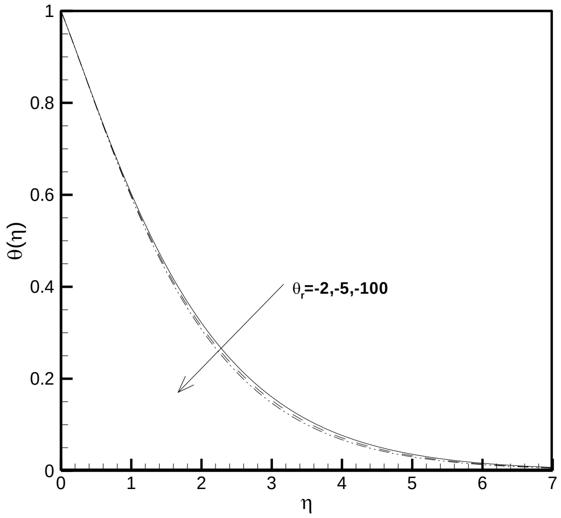

Fig. 17 The dimensionless temperature profiles for different minus values of θ_r

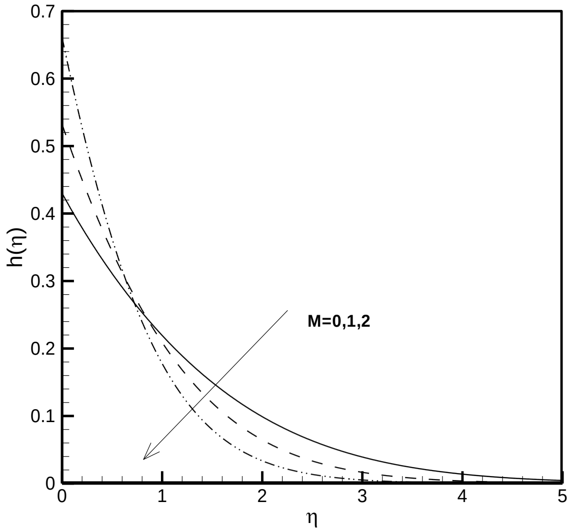

Fig. 20 The dimensionless angular velocity profiles for different values of M

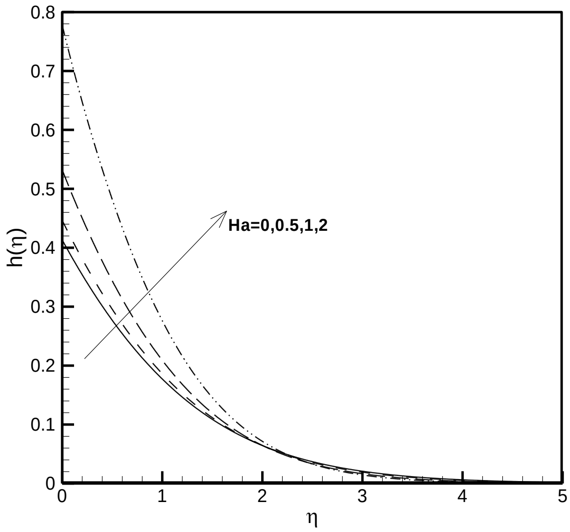

Fig. 18 The dimensionless angular velocity profiles for different values of Ha

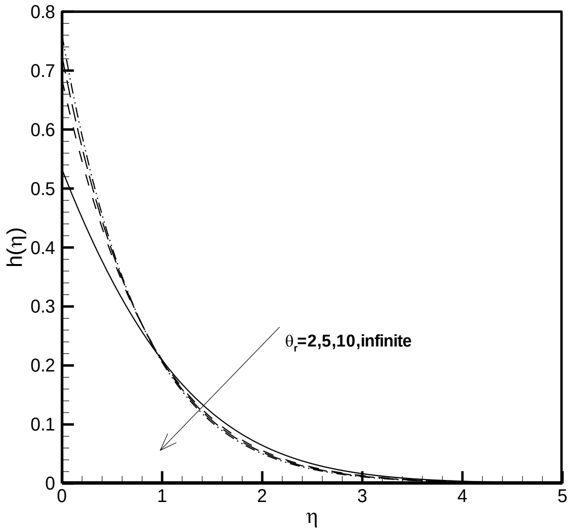

Fig. 21 The dimensionless angular velocity profiles for different positive values of θ_r

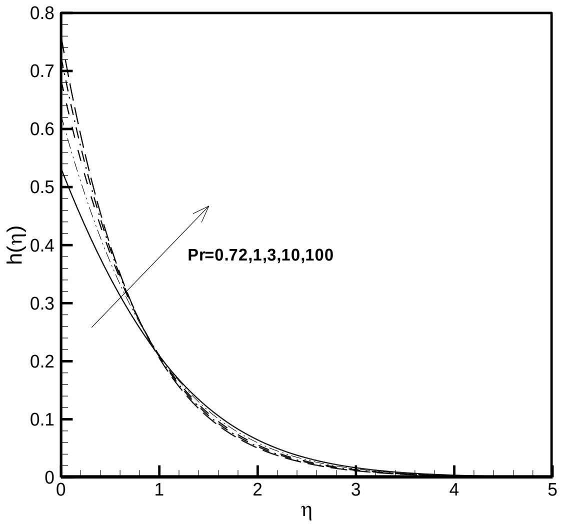

Fig. 19 The dimensionless angular velocity profiles for different values of Pr

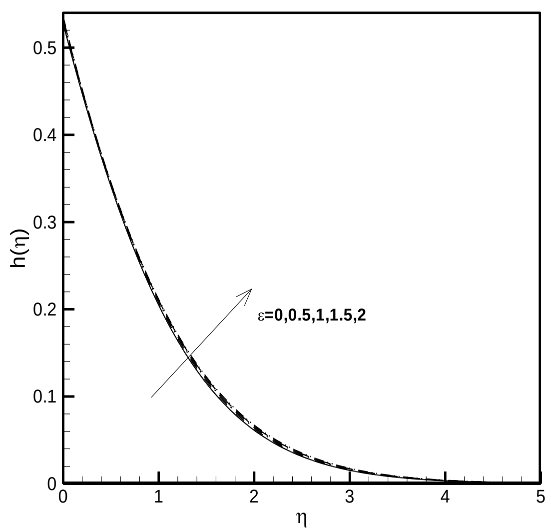

Fig. 22 The dimensionless angular velocity profiles for different values of ε

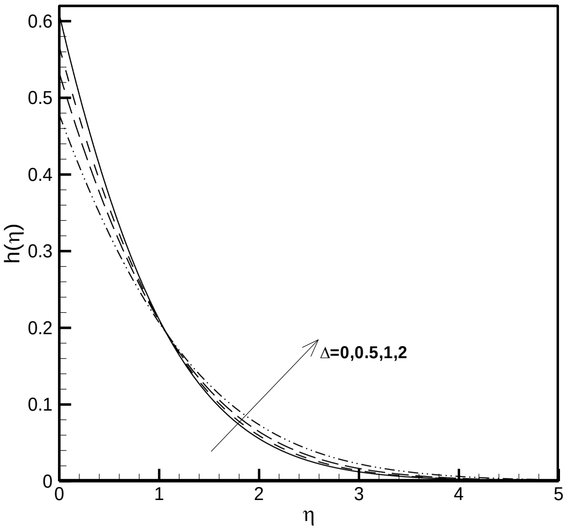

Fig. 23 The dimensionless angular velocity profiles for different values of Δ

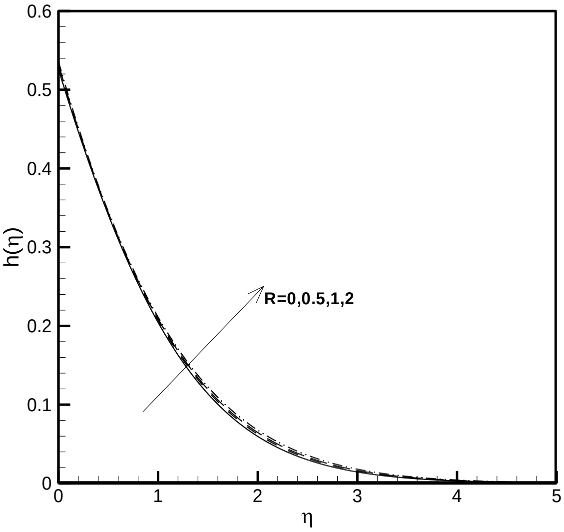

Fig. 24 The dimensionless angular velocity profiles for different values of R

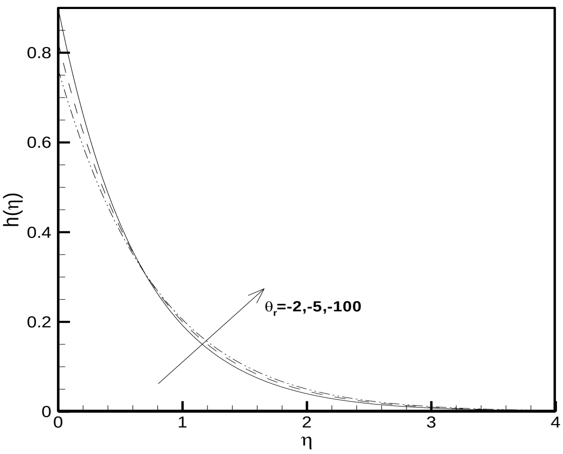

Fig. 25 The dimensionless angular velocity profiles for different minus values of θ_r

In addition, a decline in the fluid velocity is implied as a decrease of replacing heated fluid flow with the cooler flow, so the values of temperature in the thermal boundary layer augment (figures16, 17) and heat transfer rate from the sheet is suppressed.

Figures 18-25 show the variations of dimensionless angular velocity by different parameters. According to the Eq. 20-c, the dimensionless angular velocity is completely influenced by the gradient of dimensionless velocity. Therefore, an increase in the absolute values of velocity gradient leads to more values of angular velocity and vice versa. As shown in figures. 18-21, enhancement of the absolute values of velocity gradient by an increase in Ha number, Pr number, suction parameter and positive variable viscosity near the wall, leads to more magnitude of the dimensionless angular velocity profile while far from the wall, a decrease in the absolute values of velocity gradient leads to lower magnitude of angular velocity. On the other hand, figures 22-25 show the reverse effects of increasing the variable thermal conductivity, material parameter, radiation parameter and the absolute values of minus variable viscosity on the angular velocity profiles.

3. Conclusion

This research has considered the characteristics of micropolar fluid flow over a stretching sheet and heat transfer from the surface. The influences of the thermal radiation, the magnetic field, the permeability of the sheet, Prandtl number and the material parameter are considered. The dynamic viscosity and thermal conductivity coefficients are stated by credible temperature-dependent relationships, and the effects of variable viscosity parameter and variable thermal conductivity parameter are investigated. The results show that the temperature profiles increase in the presence of radiation parameter, magnetic field, variable viscosity and variable thermal conductivity parameters, while an increase in the material parameter, suction parameter and Prandtl number decreases them. Also, the results reveal that enhancement of the variable thermal conductivity, radiation and material parameters raises the values of velocity in the boundary layer, while other parameters have reverse effects on the velocity profile. Meanwhile, the angular velocity profiles are affected by the absolute values of velocity.

Nomenclature	
B	Magnetic field intensity (Tesla)
c	Stretching coefficient (s^{-1})
C_p	Specific heat ($J.kg^{-1}.°K^{-1}$)
Ec	Eckert number
f	Dimensionless steam function
H	Dimensionless angular velocity

Ha	Hartmann number
j	Microinertia (m^2)
k	Thermal conductivity of fluid ($W.m^{-1}.°K^{-1}$)
m	Microrotation parameter (rad)
M	Suction parameter
N	Angular velocity ($rad.s^{-1}$)
Pr	Prandtl number
q_r	Radiative heat flux ($W.m^{-2}$)
R	Radiation parameter
S	Vortex viscosity ($kg.m^{-1}.s^{-1}$)
T	Temperature (°K)
u	Velocity component in x-direction ($m.s^{-1}$)
v	Velocity component in y-direction ($m.s^{-1}$)
x	Horizontal coordinate (m)
y	Vertical coordinate (m)
Δ	Dimensionless material parameter
ε	Variable thermal conductivity parameter
ψ	Stream function ($m^2.s^{-1}$)
γ	Thermal properties of the fluid ($°K^{-1}$)
η	Dimensionless coordinate (Similarity variable)
κ	Absorption coefficient (m^{-1})
μ	Dynamic viscosity ($kg.m^{-1}.s^{-1}$)
ν	Kinematic viscosity ($m^2.s^{-1}$)
θ_r	Variable viscosity parameter
ρ	Density of fluid ($kg.m^{-3}$)
σ	Electrical conductivity ($Siemen.m^{-1}$)
ξ	Stefan–Boltzmann constant (W.m-2.K-4)
Subscripts	
w	At the wall
∞	Conditions far from the surface

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