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Simulation and Comparison of Non-Newtonian Fluid Models Using LBM in a Cavity

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ABSTRACT

In this paper, simulation of non-Newtonian fluid flow in a two-dimensional lid-driven cavity is investigated. In this simulation Lattice Boltzmann method is used to solve computational fluid dynamics equations numerically. The particular approach of this research is to simulate non-Newtonian fluid flow by Sisko and Hershel Bulkley extended models for the first time beside other non-Newtonian models, by means of Lattice Boltzmann technique. The results of different models including x and y-velocity profiles and streamlines are presented. Then the simulation results of different non-Newtonian fluid flow by Sisko and Hershel Bulkley extended models have been compared with Power Law, Herschel Bulkley and Bingham plastic models. Also, the effect of the Reynolds number and Power Law parameter (n) on the velocity profiles are studied. Increase of n parameter and Reynolds number leads to moving the centre of main vortex toward centre of the cavity. By increasing the parameter n, the maximum value of velocity increases and this indicates while n parameter is increased, vortex strength is excessed.

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1. Introduction

Flow simulation in a two-dimensional cavity is one of the most widely used fluid flow problems that are used to study different solution methods [1-13]. This geometry consists of a two-dimensional square with the upper side moving at a certain speed to one side. In addition to simplicity of computational space, one of the advantages of this problem is that we can easily compare the results with the results of other researchers.

In recent years, Lattice Boltzmann Method as a powerful alternative technique for fluid flow simulation particularly for complicated flows and complex boundary condition is used. compared with traditional CFD methods, Lattice Boltzmann Method, using linear relations, does the simulation procedure on lower time and lower calculation cost [1-12]. Lattice Boltzmann is a proper method for non-Newtonian simulation, because in this method strain tensor is calculated locally which improves solution stability.

Thohura et al. [14] did a numerical solution of Power Law fluid flow in a lid-driven skewed cavity. They used the two-dimensional unsteady Navier-Stokes equations in non-dimensional form. Dalal et al. [15] presented a numerical study of the flow of shear-thinning viscoelastic fluids in rectangular lid-driven cavities for a wide range of aspect ratios varying from 1/16 to 4. Mahmood et al. [16] investigated flow simulation in a single and double lid driven cavity to study the flow of a Bingham viscopelastic flow. Finite element method was used to discretize the governing equation. Furtado et al. [17] used finite element method to simulate fluid flow in the cavity to check the role of elasticity for inertialess flows of viscopelastic materials within the cavity. Sousa et al. [18] simulated the flow of viscoelastic fluids in two-dimensional cavities with a wide range of aspect ratios (height/length) 0.125 to 4. Li et al. [19] used multi-relaxation-time lattice Boltzmann method to simulate power law fluid flows in two-dimensional square cavity. Effect of the Reynolds number and Power Law parameter was investigated on the vortex properties and velocity distribution. Wang and Ho [20] simulated shear-thinning non-Newtonian blood flows with D2Q9 lattice Boltzmann Method using rheology

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models: Power Law, Carreau-Yasuda and Casson. Buick and Boyd [21] investigated the application of Lattice Boltzmann method to non-Newtonian fluid simulation in the mixing section of a screw extruder. Poursharifi and Sadeghy [22] used lattice Boltzmann method to simulate the flow of viscopelastic fluids in a closed cavity. They have numerically studied the effect of a fluid's yield stress on the single species flow driven by large amplitude peristaltic waves propagating with different phase shifts along the upper or and/or lower walls of a closed cavity.

Ashrafizadeh and Bakhshaei [23] used three non-Newtonian model, K-L, Casson, and Carreau-Yasuda to simulate blood flow with Lattice Boltzmann Method. Rahmati and Ashrafizadeh [24] simulated an incompressible fluid flow with a generalized Lattice Boltzmann Method. Bingham plastic model was used by Tang et al. [25] to simulate incompressible fluid flow by Lattice Boltzmann Method. Gokhale et al. [26] used Power Law non-Newtonian model to simulate the fluid flow in a lid-driven cavity. Perumal [27] computed the multiple solution flow properties in a double sided square and rectangular cavity. Mendu and Das [28] used the lattice Boltzmann method to study the fluid flow behaviour in a double-sided mixing cavity. Power Law as a non-Newtonian model was used in their calculations. Also, they studied the fluid flow in a cavity with oscillating lid [29]. They presented the velocity profiles for different times. Bisht and Patil [30] employed the multi-relaxation-time Lattice Boltzmann Method to simulate non-Newtonian fluid flow, modelled by Power Law, in a two-dimensional enclosure. Hussain and Huq [31] simulated the laminar, turbulent and transient flow over a two and three-dimensional cavity. They did this research in range of 1 to 7500 Reynolds numbers. Subrahmanyam and Dasp [32] simulated fluid flow in one-sided and two-sided lid-driven cavity by Lattice Boltzmann Method. Chai et al. [33] investigated differences between non-Newtonian models: Power Law, Bingham, Casson for a 2D channel and a cavity simulation. Bingham and Casson models were used by El-Borhamy [34] to simulate non-Newtonian flow in a channel and a cavity by Lattice Boltzmann Method. Other geometries also are used for investigation of application of Lattice Boltzmann Method in cavity flow simulation by Sidiki [35]. Yapici et al. [36] studied fluid flow in a two-dimensional lid-driven cavity using finite volume. They investigated the differences between Newtonian and viscoelastic fluids. Yapici and Uludag [37] presented the simulation results for three different shear-thinning fluid in a two- dimensional lid-driven cavity. Li et al. [38] considered Reynolds number and Power Law parameter influence on vortex strength and velocity distribution. They simulated fluid flow in a lid driven cavity by Multi-Relaxation Time Lattice Boltzmann Method. Madlener et al. [39] presented a generalized Reynolds number equation for Herschel Bulkley Extended, Bingham Plastic, Power Law and Herschel Bulkley. They investigated their relation endorsement by a result comparison with experimental marks. Santos et. al. [40] did a rheology study to examine effect of shear on flow curves of colloidal gels prepared with different concentration of fumed silica. Herschel Bulkley extended model was used to describe shear impact. The results were compared by experimental findings. A good agreement between the experimental and simulation results was occurred finally. Ghasemi et. al. [41] simulated two Phase mud flow, carrying cuttings, in a well using Herschel Bulkley Extended model. They used Computational Fluid Dynamics to consider the flow in rotational space between well wall and drill string. The results showed that Herschel Bulkley Extended is a proper non-Newtonian model for mud flow simulation.

Differ from the most of previous researches have been done before, Newtonian fluid simulation or non-Newtonian fluid flow modelling using Power Law model, in this work, the results of Herschel Bulkley extended and Sisko in addition to other non-Newtonian models such as Power law, Bingham plastic, Herschel Bulkley, in range of 100–1000 for Reynolds number are compared using Single Relaxation Time in Lattice Boltzmann method, that despite the acceptable accuracy it has more simplicity than other methods. More importantly, Herschel Bulkley Extended and Sisko models are used for the first time in a lid driven cavity fluid flow simulation through Lattice Boltzmann method.

2. Non-Newtonian Models

Non- Newtonian fluids refer to the fluids that has no single constitutive equation to describe exactly their relationship between the shear stress and the shear rate overall ranges of shear rates. Three major categories of non-Newtonian fluids are basically recognized, namely, time-independent, and time-dependent and viscoelastic. The time independent category has received a substantial degree of attention in comparison with the other two categories.

As an example, in conventional drilling, drilling fluids are modeled with classical rheological models like Bingham plastic or Power Law model and fluid behavior is defined with only two points of the rheological relation. These points correspond to higher shear rates. This approach can be justified in the case of conventional drilling. The knowledge of rheological data and methods of predicting pressure losses are the key points to calculate proper pump rate and avoid any obstacle in drilling operation.

Non-Newtonian fluid models equations can be written as below:

Power Law model:

$$\tau = K\dot{\gamma}^n \tag{1}$$

Sisko model:

$$\tau = \eta_\infty\dot{\gamma} + K\dot{\gamma}^n \tag{2}$$

Bingham plastic model

$$\begin{cases} \dot{\gamma} = 0 & |\tau| < \tau_y \\ |\tau| = \tau_y + \mu_p|\dot{\gamma}| & |\tau| > \tau_y \end{cases} \quad (3)$$

A rheological model that is thought to represent the flow behavior of non-Newtonian fluids very well is the Herschel Bulkley model or yield power law model. Herschel Bulkley model merges the theoretical and practical aspects of Bingham and power law models. Herschel Bulkley model:

$$\begin{cases} \dot{\gamma} = 0 & |\tau| < \tau_y \\ |\tau| = \tau_y + K|\dot{\gamma}|^n & |\tau| > \tau_y \end{cases} \quad (4)$$

Herschel Bulkley Extended model:

$$\begin{cases} \dot{\gamma} = 0 & |\tau| < \tau_y \\ |\tau| = \tau_y + K|\dot{\gamma}|^n + \eta_\infty|\dot{\gamma}| & |\tau| > \tau_y \end{cases} \quad (5)$$

η_∞ as additional term in Sisko and Herschel Bulkley Extended model is defined as constant viscosity in very high shear rate range [50].

Papanastasiou [47] suggested a new form for Bingham, Herschel Bulkley and Herschel Bulkley extended models in order to solve the Shear stress discontinuity in these models [25,48]. So Bingham Plastic model relation (3) converts to:

$$|\tau| = \tau_y(1 - e^{-m\dot{\gamma}}) + \mu_p|\dot{\gamma}| \quad (6)$$

And modified Herschel Bulkley relation (4):

$$|\tau| = \tau_y(1 - e^{-m\dot{\gamma}}) + K|\dot{\gamma}|^n \quad (7)$$

and finally Herschel Bulkley relation (5) changes to this form:

$$|\tau| = \tau_y\left(1 - e^{-m\dot{\gamma}}\right) + K|\dot{\gamma}|^n + \eta_\infty|\dot{\gamma}|. \quad (8)$$

In above equations, m is the regularization parameter or the stress growth exponent, that controls the exponential growth of the stress.

3. Lattice Boltzmann Method

Lattice Boltzmann method as aforementioned, is one of CFD methods to simulate fluid flow. This method simplifies the governing equations based on distribution function and equilibrium distribution function. This technique is divided into two steps named collision and streaming that are defined as follows:

Collision

$$\widetilde{f}_i(\boldsymbol{x}.t) = f_i(\boldsymbol{x}.t) - \frac{1}{\tau}(f_i(\boldsymbol{x}.t) - f_i^{eq}(\boldsymbol{x}.t)) \quad (9)$$

Streaming

$$f_i(\boldsymbol{x} + c_i\Delta t.t + \Delta t) = \widetilde{f}_i(\boldsymbol{x}.t) \quad (10)$$

where f_i and $\widetilde{f}_i$ denote pre- and post-collision states of distribution function respectively. f_i^{eq} is equilibrium distribution function. c_i is the velocity particle along *i*th direction and τ is single-relaxation time parameter that is related to the kinematic viscosity ν by $\frac{2\tau-1}{6}$. Direction i depends on dimensions. Prior mentioned equations can be applied for many physically conditions. So, this method can simplify any complicated problem such as non-Newtonian, multiphase flows.

In two-dimensional flows, two-dimensional nine velocity model (D2Q9) with nine discrete velocities $\mathbf{c}_i$ (i = 0, 1, 2, ..., 8) is commonly used as below [42-44]:

$$\boldsymbol{c}_i = \begin{cases} (00), \ i = 0 \\ (cos\left[(i-1)\frac{\pi}{4}\right], \\ sin\left[(i-1)\frac{\pi}{4}\right])c, \ i = 1-4 \\ (cos\left[(i-1)\frac{\pi}{4}\right] \\ , sin\left[(i-1)\frac{\pi}{4}\right])\sqrt{2}c, \ i = 5-8 \end{cases} \quad (11)$$

where $c = \frac{\Delta x}{\Delta t}$. The equilibrium distribution functions f_i^{eq} is defined as a function of local density ρ and local fluid velocity:

$$f_i^{eq} = \omega_i\rho[1 + \frac{1}{c_s^2}\boldsymbol{c_i}.\boldsymbol{u} + \frac{1}{2c_s^4}(\boldsymbol{c_i}.\boldsymbol{u})^2 - \frac{1}{2c_s^2}\boldsymbol{u}.\boldsymbol{u}] \quad (12)$$

where c_s is the lattice speed sound and ω_i is weighting factors. Assuming $c_s = c/\sqrt{3}$ for D2Q9 lattice lead to following weighting factors:

$$\omega_i = \begin{cases} 4/9, & i = 0 \\ 1/9, & i = 1-4 \\ 1/36, & i = 5-8. \end{cases} \quad (13)$$

Density and velocity can be calculated by distribution function and equilibrium distribution function:

$$\rho(\boldsymbol{x}, t) = \sum_{i=1}^{8} f_i(\boldsymbol{x}, t) \quad (14)$$

$$u(\boldsymbol{x}, t) = \frac{1}{\rho}\sum_{i=1}^{8} c_i f_i \quad (15)$$

There are some aproach in Lattice Botzmann Method to simulate non-Newtonian fluids behaviour. One of the most important and applicable methods is peresented as below.

The $S_{\alpha\beta}$ can be taken as: [45-46]

$$S_{\alpha\beta} = -\frac{3}{\tau}\sum_{i=1}^{8} f_i^{neq}\boldsymbol{c_{i\alpha}c_{i\beta}} \quad (16)$$

Where $f_i^{neq} = f_i - f_i^{eq}$. And D_{II}, the second invariant of the strain rate tensor is defined as:

$$D_{II} = \sum_{\alpha,\beta=1}^{l} S_{\alpha\beta}S_{\alpha\beta} \quad (17)$$

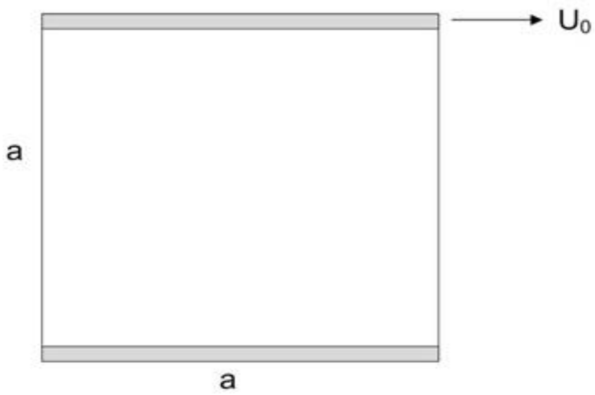


Figure 1. Geometry and boundary conditions of lid driven cavity.

The parameter l is related in dimension. For a 2-D simulation l is 2. $\dot{\gamma}$ is equal to $2\sqrt{D_{II}}$ and can be used in relations (1)–(8) in working non-Newtonian model. It means that for example for calculating the results of Power Law model, after D_{II}, $\dot{\gamma}$ can be calculated and then the using of Equation (1) shear stress is obtained.

In Lattice Boltzmann Method, Some models are suggested by researchers to solve instability problems in non-Newtonian models. one of the most important models is presented by He and Lou [49]. According to He-Lou incompressible Lattice Boltzmann model P^{eq} is given by:

$$P_i^{eq} \equiv c_s^2 . f_i^{eq} \tag{18}$$

So the Collision equation (9) varies to following relation:

$$\widetilde{P}_i(\boldsymbol{x},t) = P(\boldsymbol{x},t) - \frac{1}{\tau}(P_i(\boldsymbol{x},t) - P_i^{eq}(\boldsymbol{x},t)) \tag{19}$$

That velocity and pressure are computed by this equation:

$$P = \sum_i P_i \quad , \quad P_0\boldsymbol{u} = \sum_i \boldsymbol{c_i P_i} \tag{20}$$

Where P_0, constant pressure, is equal $\boldsymbol{c_s^2 \rho_0}$. In this model density is replaced by pressure, as an independent variable, to solve instability caused by local density variation that is affected by local non-Newtonian viscosity.

4. Lid -Driven Cavity

A laminar viscous flow in a square cavity which top wall moves with uniform velocity is chosen to compare the results of different non-Newtonian models. Boundary conditions and physical model of this cavity can be seen in Figure 1.

5. Validation

To check validity results, the Power Law non-Newtonian model, considering n = 1.0, is used. The results are compared with Ghia et al. [13]. In Figures 2 and 3, u-velocity and v-velocity in y and x direction for Reynolds number 100 and 400 at the midpoints respectively are presented. These calculations have been done for 129 × 129 grid in the computational domain as the Ghia et al. research. The velocity profile for Lattice Boltzmann method and Ghia et al. are obviously matched.

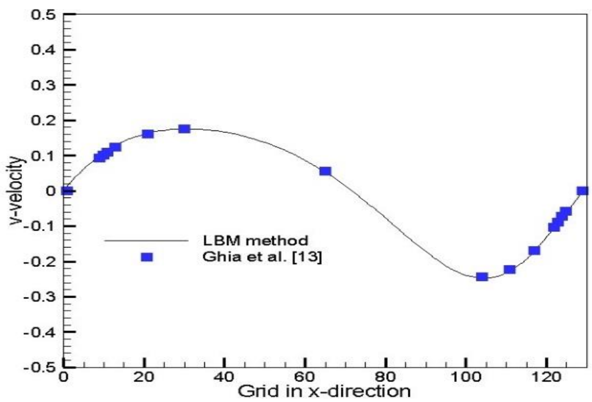
Although convergence criterion is assumed 10^{-9}, in order to correct results achievement, maximum number of iteration considered 2×10^6.

6. Lid -Driven Cavity

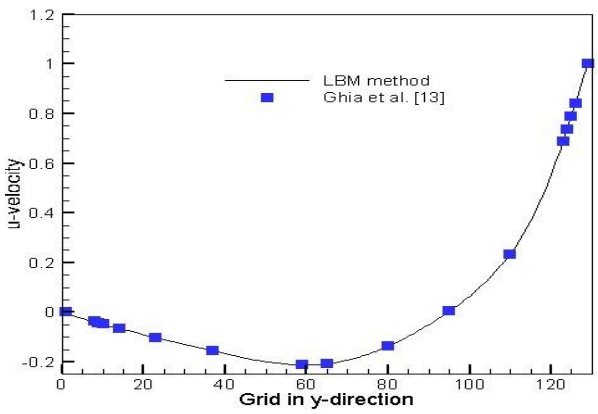
As it is shown in Figure 4, the u and v velocity in y and x-direction respectively is compared in midpoint for 5 non-Newtonian models: Power Law, Herschel Bulkley, Herschel Bulkley Extended, Sisko and Bingham. The Power Law parameter n and Reynolds number are considered 1.1 and 500.

laminar viscous flow in a square cavity which top wall moves with uniform velocity is chosen to compare the results of different non-Newtonian models. Boundary conditions and physical model of this cavity can be seen in Figure 1.

Velocity profiles in different non-Newtonian models follow the same pattern. However the maximum and the minimum values in horizontal and vertical velocity for different models differs. Also, the above results for Reynolds number 1000 and index n = 1.5 is presented in Figure 5.

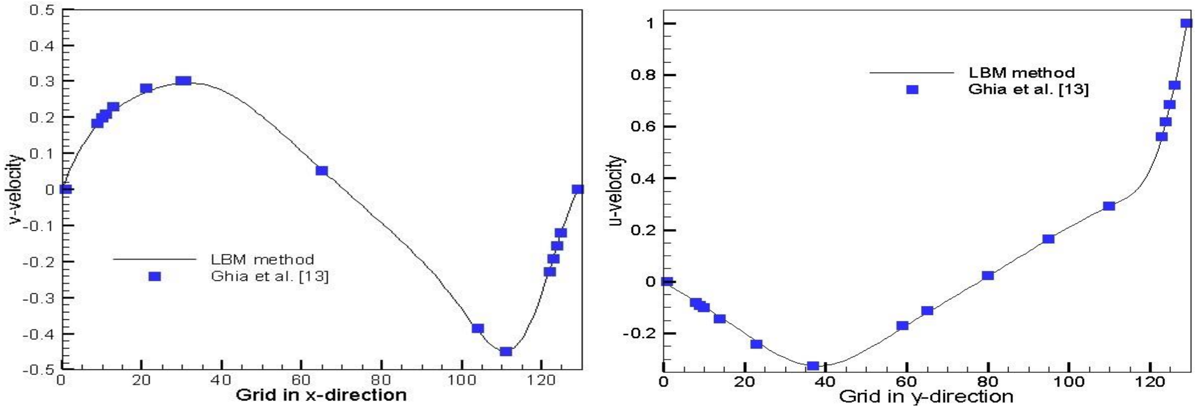


(a)



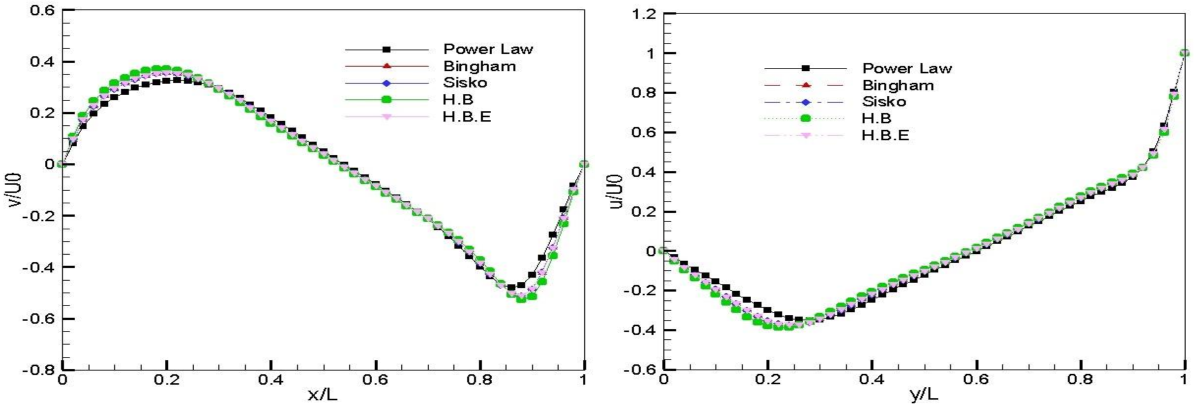
(b)

Figure 2. Comparison (a) v-velocity (b) u-velocity variation in Power Law model with Ghia et al. at Reynolds number 100.



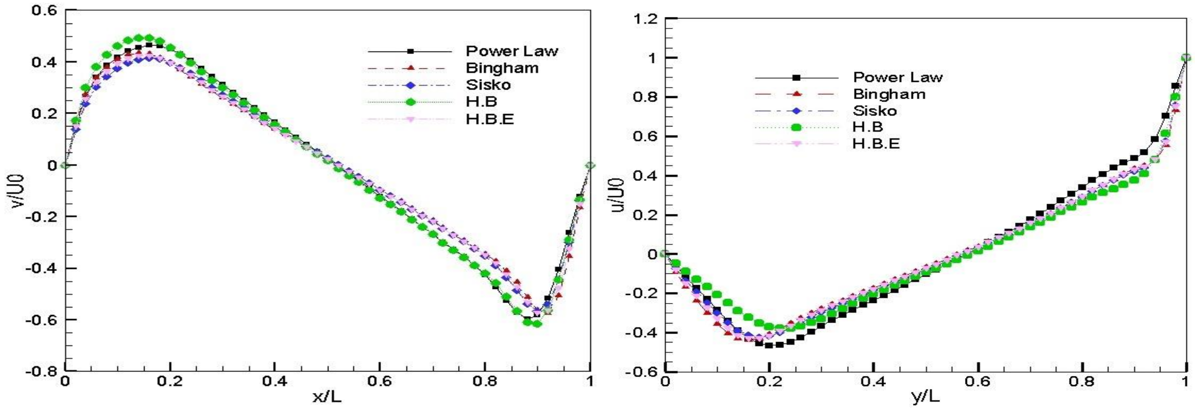
(a) (b)

Figure 3. Comparison (a) v-velocity (b) u-velocity variation in Power Law model with Ghia et al. at Reynolds number 400.



(a) (b)

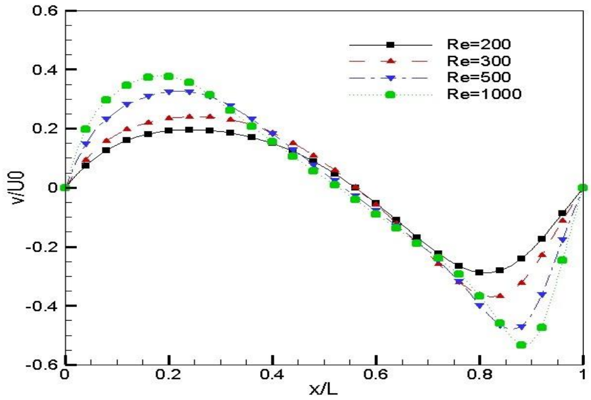
Figure 4. (a) v-velocity (b) u-velocity variation for non-Newtonian models at Reynolds number 500.



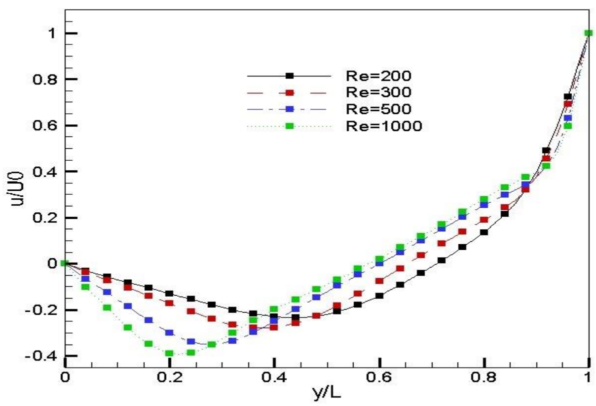
(a) (b)

Figure 5. (a) v-velocity (b) u-velocity variation for non-Newtonian models at Reynolds number 1000.

The variation of the Reynolds number in Power Law model is investigated in Figure 6. For this purpose, the results for Reynolds numbers 200, 300, 500, 1000 are compared. Increasing Reynolds number leads to larger wakes and more difference between velocities at two cavity sides and then maximum velocity increases.



(a)



(b)

Figure 6. (a) v-velocity (b) u-velocity variation for Reynolds number and power Law model**.**

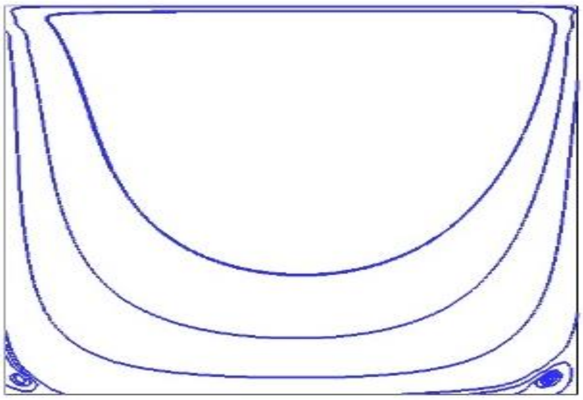
Table 1. comparison of Location of main vortex centre for Power Law model

n parameter	location	Re=100	
		Neofytou [51]	This research
n=0.5	x/H	0.7166	0.7101
	y/H	0.7804	0.7853
n=1.0	x/H	0.6123	0.6097
	y/H	0.7359	0.7388
n=1.5	x/H	0.5647	0.5702
	y/H	0.7240	0.7212

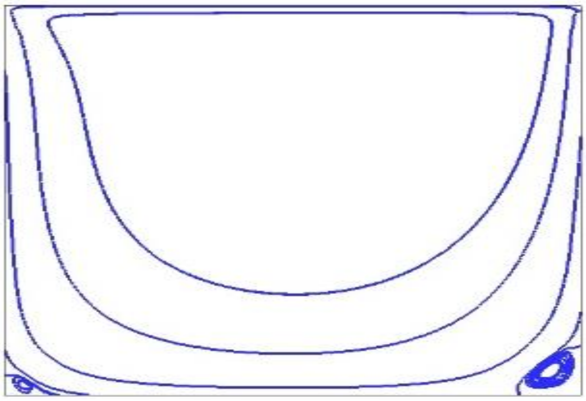
Table 2. Location of main vortex centre for Sisko model

n parameter	location	Re=100	Re=400	Re=1000
n=0.5	x/H	0.6354	0.6112	0.5733
	y/H	0.755	0.7231	0.6622
n=1.0	x/H	0.6146	0.5966	0.5538
	y/H	0.738	0.6011	0.565
n=1.5	x/H	0.5912	0.5795	0.5285
	y/H	0.7148	0.6989	0.5734

Location of centre of main vortex for different index n, in Power Law model is achieved. These results are validated by comparison with Neofytou [51]. Table 1 shows the matching and that is evident that centre of the main vortex moves toward cavity centre whereas n parameter is increasing. Streamlines in relation with table 1 items are presented in figure 7.



(a)



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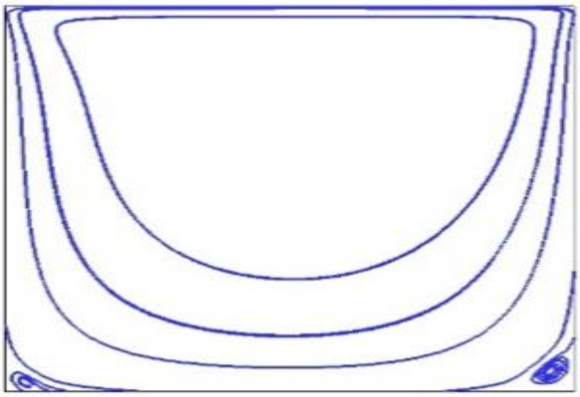
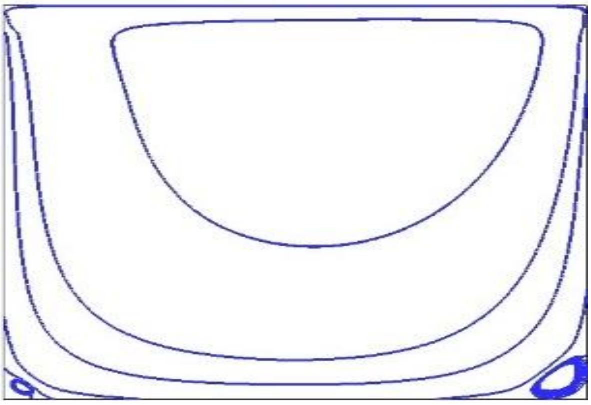


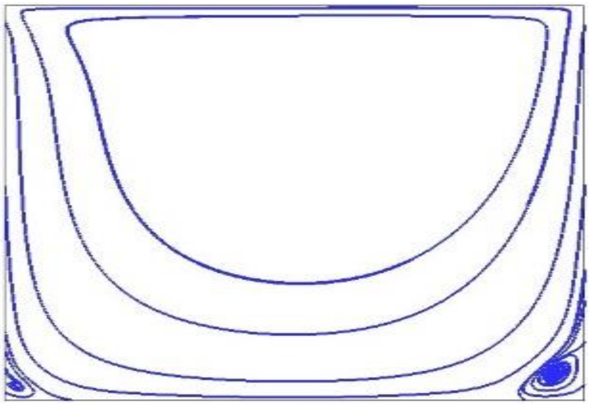
(c)

Figure 7. Streamlines in 2-D Cavity Power Law model (a) n = 0.5; (b) n = 1.0; (c) n = 1.5

Power Law parameter, n, and Reynolds number impact on location of main vortex centre in Sisko model is also investigated. Location and streamlines are presented in table 2 and figure 8, respectively. As expected, increasing n parameter leads to moving centre of main vortex toward cavity centre. It should be noted that increasing of Reynolds number affects in the same way.



(a)



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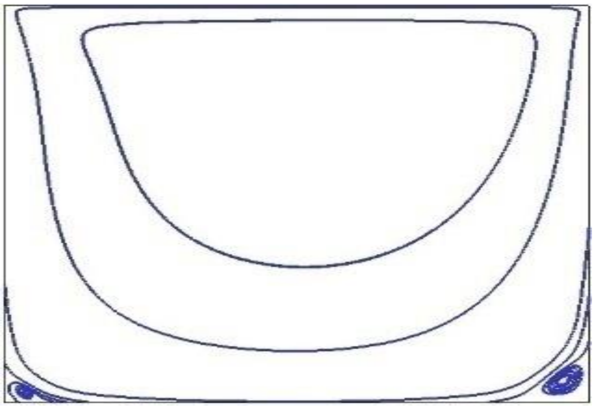
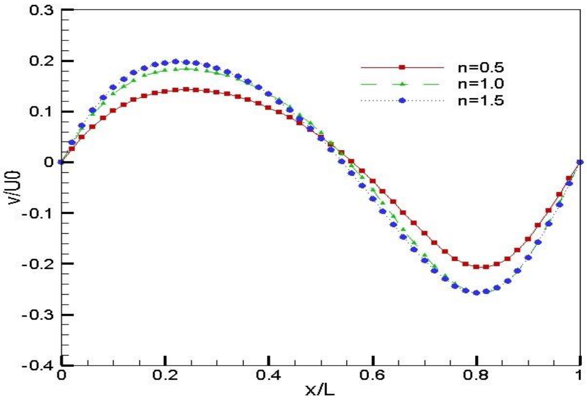


(c)

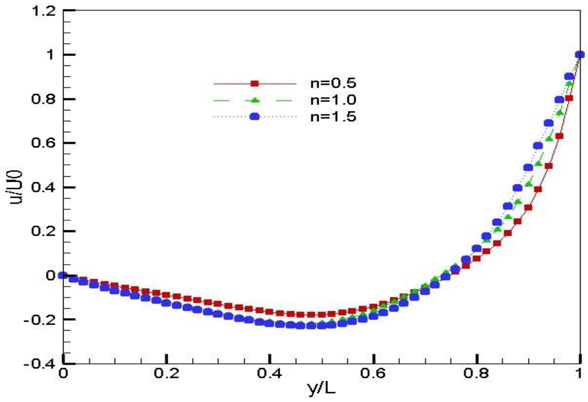
Figure 8. Streamlines in 2-D Cavity Sisko model (a) n = 0.5; (b) n = 1.0; (c) n = 1.5

Velocity in x and y-direction for non-Newtonian flow simulation in Sisko model and Reynolds number 100 and n = 0.5, 1.0 and 1.5 are presented in Figure 9. By increasing the parameter n, the maximum value of velocity increases and this indicates that increasing of n, leads to increasing vortex power.

Also, in Figures 10–12, there are streamlines in the cavity for Power Law, Herschel Bulkley and Bingham Plastic models. In these figures effect of Reynolds number increasing is obvious. The Reynolds numbers increasing leads to larger wakes in corners. Three type of vortex shedding are observed in the flow diagrams of Figure 10–12. The main vortex and the biggest vortex shedding are

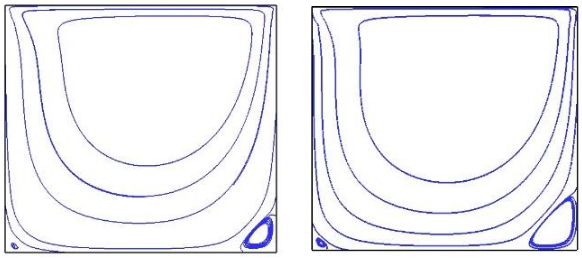


(a)

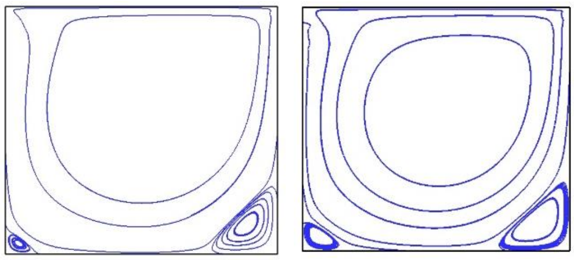


(b)

Figure 9. (a) v-velocity (b) u-velocity for various n in Sisko method.



(**a**) (**b**)



(**c**) (**d**)

Figure 10. Streamlines in 2-D Cavity Power law model (a) Re = 200; (b) Re = 300; (c) Re = 500; (d) Re = 1000.

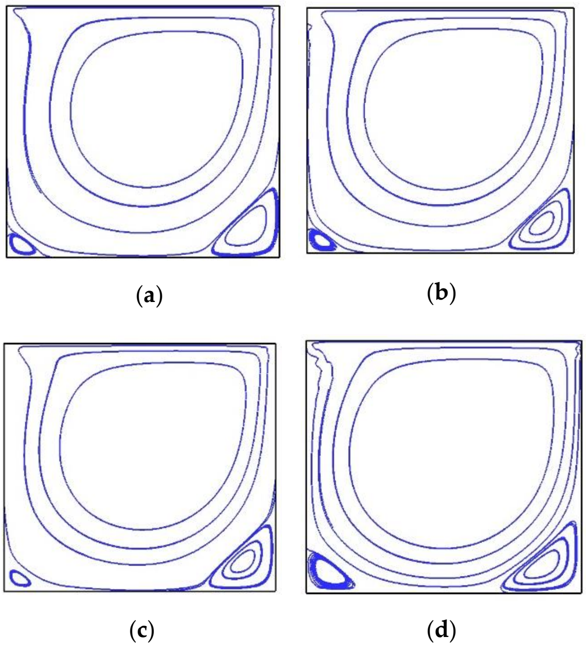


Figure 11. Streamlines in 2-D Cavity Bingham Plastic. (a) Re=200, (b) Re = 300, (c) Re = 500, (d) Re = 1000.

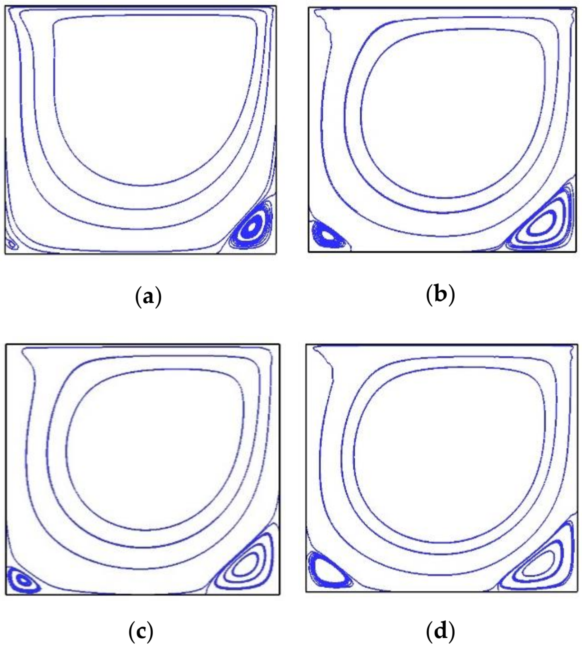


Figure 12. streamlines in 2-D cavity Herschel Bulkley model. (a) Re = 200, (b) Re = 300, (c) Re = 500, (d) Re = 1000.

visible in the centre of the cavity. And the others lead to the bottom of the cavity, on the sides of that. Of course, the other vortex will be created in much higher Reynolds numbers, on the left corner of the moving plane.

Conclusion

This study investigates Sisko and Herschel Bulkley Extended non-Newtonian models that are used for fluid behaviour simulation in addition to Power Law, Herschel Bulkley and Bingham Plastic models, using Lattice Boltzmann method. Accordingly, simulation results are compared to each other. The results, containing x-velocity, y-velocity profiles and streamlines are presented for different Reynolds numbers and different Power Law parameter. Afterwards change effect of Reynolds number and parameter n is investigated. One of the most important effects of increasing the Reynolds number can be demonstrated in streamlines. In this way, increase of Reynolds number leads to emerge new vortexes in the cavity corners. The simulation results illustrate that when the Reynolds number keeps increasing, the vortexes strengthen. Following a similar trend, enhancing the value of the n parameter leads to the maximum value of velocity growth and correspondingly the vortex power rises. In the case of fixed Reynolds number, increasing Power Law parameter causes to main vortex moves to centre of the cavity. Likewise, for a constant n, while the Reynolds number grows, it influences the location of main vortex centre and relocates its movements in the vicinity of the cavity centre. Finally, based on calculations and simulations it can be declared that in spite of the simplicity of Lattice Boltzmann method to simulate complicated fluid flow, the result of this techniques in comparison to other classical techniques depict that this method is immensely accurate and practical.

Nomenclature

C_i	Lattice speed (m/s)
c_s	Lattice speed sound (m/s)
D	second invariant of the strain rate tensor
f_i	distribution function
$\tilde{f}_i$	Density distribution function
f_i^{eq}	Equilibrium density distribution function
k	flow consistency index (W/m.k)
m	Papanastasiou parameter
n	Power Law index (dimensionless)
P	Pressure
Sα´β	Strain rate tensor (s^{-1})
t	time (s)
u	x-direction Velocity component (m/s)
v	y-direction Velocity component (m/s)
U	inflow average velocity (m/s)
Re	Reynolds number (dimensionless)
X	Cartesian coordinate (m)
Y	Cartesian coordinate (m)
$\dot{\gamma}$	Shear Strain rate (1/s)
η_∞	Herschel Bulkley Extended parameter
μ_0	viscosity (Kg/m.s)
ρ	Density(kg/m^3)
τ	Shear stress
τ_y	Yield stress
ω_i	Weighting factor

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