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Flow, Heat and Mass Transfer past a Stretching Sheet with Temperature Dependent Fluid Properties in Porous Medium

Hemanta Konwar *

Kohima Science College, Jotsoma, Nagaland, INDIA.

P A P E R   I N F O

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A B S T R A C T

An investigation is carried out to study MHD boundary layer flow, heat and mass transfer of an incompressible viscous fluid past a continuously moving nonlinear stretching porous sheet in porous medium with temperature dependent fluid properties subject to heat source, viscous dissipation, chemical reaction and suction. The fluid viscosity and the thermal conductivity are assumed to vary as an inverse function and linear function of temperature respectively. The governing nonlinear partial differential equations are converted into a system of coupled nonlinear ordinary differential equations by using similarity transformations and solved numerically by the Matlab's built in solver bvp4c. The numerical results are presented graphically for velocity, temperature and concentration distributions. The skin friction, wall temperature and wall concentration gradients are tabulated for emerging parameters. It is arrived at a good agreement on comparing the present numerical results with previously published results. It is found that skin friction rises but wall temperature and wall concentration gradients fall with growing viscosity, magnetic field, stretching parameter and Darcy parameter respectively. The thermal conductivity parameter diminishes wall temperature gradient while the Schmidt number and chemical reaction parameter enhance wall concentration gradient.

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1. Introduction

The study of boundary layer flow past a continuously moving surface is an important research area because of its applications in polymer extrusion, metallurgical, glass fiber and paper production processes, thinning and annealing of copper wires and so forth where the end product relies on the rate of cooling. Sakiadis [1,2] was the first to investigate boundary layer flow behaviour on continuous solid surface. Erickson et al. [3] studied transfer of heat and mass in laminar boundary layer on a continuously moving flat plate with suction and injection. Tsou et al. [4] studied analytically and experimentally flow and heat transfer in the boundary layer of continuously moving surface. After that Crane [5], Gupta and Gupta [6], Soundalgekar and Murty [7], Grubka and Bobba [8], Vajravelu[9], Kumari et al. [10], Gorla and Sidawi [11], Anjalidevi and Kayalvizhi [12], Malvandi et al. [13], pop et al. [14] investigated different aspects of boundary layer behaviour on a continuously moving surface. All these studies were carried out with uniform viscosity and thermal conductivity.

But later, it was found that physical characteristics such as viscosity and thermal conductivity are known to fluctuate dramatically with temperature. Investigations performed by Herwig and Wickern [15], Lai and Kulacki [16], Pop et al. [17], Chiam [18], Kumari [19], Mukhopadhyay et al. [20], El-Aziz [21], Sharma and Singh [22], Prasad et al. [23], Anjali Devi and Prakash [24] showed that consideration of temperature dependent fluid properties are necessary to predict heat and mass transfer rate accurately. Because, when the temperature is raised from 10^0C ($\mu = 0.0131\, g.cm^{-1}s^{-1}$) to 50^0C ($\mu = 0.00548\, g.cm^{-1}s^{-1}$), the viscosity of water drops by around 240% (El-Aziz [21]) and for liquid metals, the

*Corresponding Author: Hemanta Konwar
Email: hemanta.konwar@gmail.com

thermal conductivity varies with temperature in an approximately linear manner in the range from 0^0F to 400^0F(Kays[25]).

A porous medium is a material that has pores or void spaces. Materials like rocks, wood, soil, sand etc. can be considered as porous medium. Porosity and permeability are the most common characteristic of a porous material. Porosity means the quality or state of being porous and permeability measures the ease with which liquids, gases, or certain compounds may move through a substance. The study of porous medium is useful in explaining many phenomena of transport processes. The most significant area that depends on the properties of porous media is hydrology. The process of heating or cooling can be controlled by porous medium. In recent years, due attention has been given to transport processes in porous medium because of its relevance to petroleum technology, bio-chemical engineering, geophysics, cooling of nuclear reactors and other fields. The flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation or absorption and suction or blowing has been studied by Cortell[26]. Mukhopadhyay and Layek [27] studied effects of variable viscosity on flow past a heated stretching sheet embedded in a porous medium in presence of heat source or sink. Detailed literature review can be found in the books by Pop and Ingham [28], Ingham and Pop [29], Vafai [30] and Nield and Bejan [31].

When there is a large temperature variation between the surface and the ambient fluid, the heat source or sink effects in thermal convection become important. Boundary layer flow and heat transfer of a fluid with variable viscosity over a stretching sheet in a porous medium by taking into account of viscous dissipation and heat source or sink in the presence of uniform magnetic field was analyzed by Dessie and Kishan [32] and reported that viscous dissipation acted like an energy source, which affected the heat transfer rates. The effects of viscous dissipation and variable physical properties on steady natural convection heat and mass transfer flow through a vertical channel were investigated by Ojeagbase and Ajibade [33].

Heat production is also significant in chemical reactions that are either exothermic or endothermic. Considerable interest is growing on studying chemical reaction effects on boundary layer flow as chemical reaction involves in making and breaking of bonds in the reactive substances and its effects also determine heat and mass transfer rates. It has also practical applications in electrochemistry and chemical engineering. Damseh et al. [34] discussed effects of heat source or sink and first order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Bhattacharyya et al. [35] considered MHD boundary layer stagnation point flow over a stretching sheet with chemical reaction. Patil and Chamkha [36] examined transfer of heat and mass in a mixed convection flow of polar fluid along a plate in porous media subject to chemical reaction. Mjankwi et al. [37] investigated unsteady MHD flow of nanofluid with variable properties over a stretching sheet in the presence of thermal radiation and chemical reaction.

The problem of heat and mass transfer of a hydromagnetic boundary layer flow past a continuously moving nonlinear stretching permeable sheet in porous medium with temperature dependent viscosity and thermal conductivity subject to heat source, viscous dissipation, chemical reaction and suction is unexplored. Therefore, a study is conducted to analyse the hydromagnetic boundary layer flow, heat and mass transfer of an electrically conducting and chemically reacting incompressible viscous fluid past a continuously moving nonlinear stretching permeable sheet in porous medium with temperature dependent fluid properties subject to heat source, viscous dissipation, chemical reaction and suction.

2. Mathematical Formulation

Consider a steady, two-dimensional, laminar, electrically conducting and chemically reacting incompressible viscous fluid flow over a stretching permeable sheet coinciding to the plane $y = 0$ in porous medium. The origin O is located at the slit. The x-axis is taken along the sheet in the direction of the flow and the y-axis is perpendicular to it. Let two equal and opposite forces are applied along the x-axis, keeping the origin O fixed so that the wall is stretched continuously in the direction of the flow with a power law velocity $u = u_w(x) = ax^n$, where a is the stretching rate, n is the exponent such that $n > 0$ for accelerated sheet and $n < 0$ for decelerated sheet from the extruded slit, and x measures the distance from the origin O. Let a variable transverse magnetic field of strength $B(x) = B_0 x^{(n-1)/2}$ is applied (Chiam [18]), where B_0 is the magnetic field intensity. Further, the heat source and chemical reaction are thought to be of the form $Q(x) = Q_0 x^{(n-1)}$ and $k_c(x) = k_0 x^{(n-1)}$ where Q_0 and k_0 are the heat generation coefficient and the local chemical reaction coefficient respectively. These forms are used here to obtain a similarity solution. The induced magnetic field is being ignored as the magnetic Reynolds number is assumed to be smaller in magnitude. The effects of Joule heating and Hall current are also assumed to be insignificant.

With these assumption the governing equations, which models the flow can be written as (Prasad et al. [23]).

The Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The Momentum Equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B^2(x)}{\rho_\infty}u - \frac{\mu u}{\rho_\infty k_p} \quad (2)$$

The Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho_\infty c_p}\left\{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)\right\}+\frac{1}{\rho_\infty c_p}Q(x)(T-T_\infty)+\frac{\mu}{\rho_\infty c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

The Species Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_c(x)(C-C_\infty) \quad (4)$$

with the boundary conditions:

$$u=u_w(x)=ax^n,\ \ v=v_w,\ \ T=T_w,\ \ C=C_w \text{ at } y=0 \quad (5)$$

and

$$u\to 0,\ \ T\to T_\infty,\ \ C\to C_\infty \ \text{ at } y\to\infty \quad (6)$$

where u and v are the velocity components along the x and y-axes, respectively, σ is the electrical conductivity, T, C are the temperature and concentration of the fluid in the boundary layer, μ is the temperature dependent viscosity, ρ_∞ is the constant fluid density, c_p is the specific heat at constant pressure, T_w, C_w are the temperature and concentration on the sheet, D is the coefficient of mass diffusivity, k is the temperature dependent thermal conductivity of the fluid, v_w is the wall velocity normal to the sheet and k_p is the permeability of the porous medium respectively. Assume that the fluid viscosity μ is an inverse function of temperature T (Lai and Kulacki [16]) and the thermal conductivity k is a linear function of temperature T(Slattery [38]) respectively, thus:

$$1/\mu=[1+\delta(T-T_\infty)]/\mu_\infty \quad (7)$$

or

$$1/\mu=b(T-T_r), b=\delta/\mu_\infty,\ T_r=T_\infty-1/\delta, \quad (8)$$

and

$$k=k_\infty\{1+c(T-T_\infty)\} \quad (9)$$

where T_r is the transformed reference temperature corresponding to viscosity parameter, b and δ are constants that depend on the reference state and thermal property of the fluid, μ_∞ and k_∞ are the viscosity and the thermal conductivity of the fluid far away from the wall, c is a constant depends on nature of the fluid. In general, $b>0$ for liquid, $b<0$ for gases and $c>0$ for fluids such as water and air while, $c<0$ for fluids such as lubricating oils. The correlation between viscosity and temperature of air and water which are mostly used fluids in industries are given as follows (Lai and Kulacki [16]) :
for air: $1/\mu=-123.2(T-742.6)$,
based on $T_\infty=293K(20^0C)$
and for water: $1/\mu=29.83(T-258.6)$,
based on $T_\infty=288K(15^0C)$

Introducing the following transformations:

$$\left.\begin{array}{c}\eta=\sqrt{a(n+1)/2\nu_\infty}\,x^{(n-1)/2}y,\\ \psi(x,y)=\sqrt{2\,a\nu_\infty/(n+1)}\,fx^{(n+1)/2},\\ \theta(\eta)=(T-T_\infty)/(T_w-T_\infty)\\ and\ \phi(\eta)=(C-C_\infty)/(C_w-C_\infty)\end{array}\right\} \quad (10)$$

into the equations Eq.(1) to Eq.(9) it is obtained that:

$$\mu=\mu_\infty\,\theta_r/(\theta_r-\theta), \quad (11)$$

$$k=k_\infty(1+\varepsilon\theta), \quad (12)$$

$$\frac{\theta_r}{\theta_r-\theta}\{f'''-2\,Re\,f'/(n+1)Da\}+\frac{\theta_r}{(\theta_r-\theta)^2}f''\theta'-2Mf'/(n+1)+ff''-\beta(f')^2=0, \quad (13)$$

$$(1+\varepsilon\theta)\theta''+\varepsilon\left(\theta'\right)^2+2PrS\theta/(n+1)+\frac{\theta_r}{\theta_r-\theta}EcPr(f'')^2+Prf\theta'=0, \quad (14)$$

$$\phi''+Scf\phi'-2ScN\phi/(n+1)=0 \quad (15)$$

with the transformed boundary conditions:

$$f=\lambda,\ f'=1, \theta=1, \phi=1 \text{ at } \eta=0 \quad (16)$$

and

$$f'\to 0, \theta\to 0, \phi\to 0 \ \text{ at } \eta\to\infty \quad (17)$$

Here prime represents differentiation with respect to η. It is clear that the stream function ψ where $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ identically satisfies the Continuity Equation Eq. (1).

In the equations Eq. (13) to Eq. (15) the non-dimensional parameters:
$Re=u_w x/\nu_\infty$, $Da=u_w^2 k_p/\nu_\infty^2$,
$M=\sigma B_0^2/u_0\rho_\infty$, $\beta=2n/(n+1)$,
$Pr=\mu_\infty c_p/k_\infty$,
$S=Q_0/\rho_\infty c_p u_0$,
$Ec=u_w^2/c_p\,(T_w-T_\infty)$,
$Sc=\nu_\infty/D$ and $N=k_0/u_0$ are Reynolds number, Darcy parameter, magnetic parameter, stretching parameter, Prandtl number, heat source or heat generation parameter, Eckert number or viscous dissipation parameter, Schmidt number and chemical reaction parameter respectively. The thermal conductivity of the fluid is $\varepsilon=c(T_w-T_\infty)$. The range of variations of ε can be taken as: $0\le\varepsilon\le 6$ for air, $0\le\varepsilon\le 0.12$ for water and $-0.1\le\varepsilon\le 0$ for lubricating oils (Schilichting [39]). $\beta>0$ for stretching sheet, $\lambda=-v_w\sqrt{2/a\nu_\infty(n+1)}\,x^{(1-n)/2}$ is the suction or injection parameter according as $v_w<0$ or $v_w>0$ and $S>0$ for heat source.

The equation Eq.(7) can be rewritten as $\mu=\mu_\infty/\{1-(\theta/\theta_r)\}$. As θ varies from 0, at the leading edge of the boundary layer, to 1 at the surface of the sheet, the highest change in fluid viscosity from its free stream value, μ_∞, occurs at the sheet surface where $\mu_w = \mu_\infty/(1-\theta_r^{-1})$ (El − Aziz [21]). It is to be noted that $(-\theta_r) \to \infty$ leads to $\mu \to \mu_\infty$ i.e the variation of viscosity in the boundary layer is negligible but as $(-\theta_r) \to 0$ the variation of viscosity becomes more remarkable. It is worth to be mentioned that the viscosity parameter $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{1}{\delta(T_w - T_\infty)}$ is negative for liquids and positive for gases whenever $T_w - T_\infty$ is positive. In the present investigation θ_r is assumed negative.

The parameters of engineering importance in transfer of heat and mass are the local skin friction coefficient C_f, the local Nusselt number Nu and the local Sherwood number Sh. These parameters describe the surface drag, wall heat and mass transfer rates respectively. The local skin friction coefficient is given by:

$$C_f = \frac{2\tau_w}{\rho_\infty u_w^2} \quad \text{where}$$

$$\tau_w = -\mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = -\sqrt{2(n+1)}\,\theta_r/(\theta_r - 1)\ Re^{-1/2} f''(0) \tag{18}$$

The Nusselt number is given by:

$$Nu = \frac{x q_w}{k_\infty (T_w - T_\infty)} \quad \text{where}$$

$$q_w = -k_\infty \left.\frac{\partial T}{\partial y}\right)_{y=0} = -\sqrt{(n+1)/2}\, Re^{1/2} \theta'(0) \tag{19}$$

The Sherwood number is given by:

$$Sh = \frac{x m_w}{D(C_w - C_\infty)} \quad \text{where}$$

$$m_w = -D \left.\frac{\partial C}{\partial y}\right)_{y=0} = -\sqrt{(n+1)/2}\, Re^{1/2} \phi'(0) \tag{20}$$

3. Numerical Method for Solution

To obtain numerical results, the system of ordinary differential equations Eq.(13) to Eq.(15) subject to the boundary conditions equations Eq.(16) and Eq.(17) are converted into a system of first order ordinary differential equations and then solved by employing Matlab's built in solver bvp4c (Shampine et al.[40]) with some initial guesses. The process is shown below. Setting:

$$f = y_1, f' = y_2, f'' = y_3, \theta = y_4, \theta' = y_5, \phi = y_6 \text{ and } \phi' = y_7 \tag{21}$$

and by using the equation Eq. (21) into the equations Eq. (13) to Eq. (15) the following system of first order ordinary differential equations Eq. (22) to Eq. (28) together with boundary conditions equations Eq. (29) and Eq. (30) can be obtained.

$$y_1' = y_2 \tag{22}$$

$$y_2' = y_3 \tag{23}$$

$$y_3' = \frac{(\theta_r - y_4)}{\theta_r}\{2My_2/(n+1) + \beta y_2^2 - y_1 y_3\} - \frac{y_3 y_5}{\theta_r - y_4} + 2Re y_2/\{(n+1)Da\} \tag{24}$$

$$y_4' = y_5 \tag{25}$$

$$y_5' = -\frac{1}{(1+\varepsilon y_4)} \times \left\{\varepsilon y_5^2 + 2PrSy_4/(n+1) + Pr y_1 y_5 + \frac{\theta_r Pr Ec}{(\theta_r - y_4)} y_3^2\right\} \tag{26}$$

$$y_6' = y_7 \tag{27}$$

$$y_7' = -Sc\{y_1 y_7 - 2Ny_6/(n+1)\} \tag{28}$$

$$y_1(0) = \lambda, y_2(0) = 1, y_4(0) = 1, y_6(0) = 1 \tag{29}$$

$$y_2(\infty) = 0, y_4(\infty) = 0, y_6(\infty) = 0 \tag{30}$$

4. Results and Discussions

In this paper hydromagnetic boundary layer flow, heat and mass transfer past a continuously moving nonlinear stretching permeable sheet in a porous medium with temperature dependent viscosity and thermal conductivity subject to heat source, viscous dissipation, chemical reaction and suction are studied. The fluid viscosity and the thermal conductivity are assumed to vary as an inverse function and linear function of temperature respectively. The governing nonlinear partial differential equations are converted into a system of coupled nonlinear ordinary differential equations by utilizing suitable similarity transformations and then solved by bvp4c. The numerical results are presented graphically for velocity, temperature and concentration distributions and the behaviour of various emerging parameters are analyzed.

For numerical calculations, various parameter values are considered as follows:

$\varepsilon = 0.1$; $\beta = 1$; $\theta_r = -5$; $Re = 1$;

$M = 1$; $n = 0.2$; $Da = 0.5$; $Pr = 7$;

$S = 0.2$; $Ec = 0.1$; $Sc = 0.62$; $N = 0.5$;

$\lambda = 1$. unless stated otherwise.

Figures 1-6 represent the effects of viscosity parameter θ_r and magnetic parameter M on horizontal velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ distributions. It is found that both viscosity parameter θ_r and magnetic parameter M

reduce the momentum boundary layer but boost the concentration boundary layer. It is because higher the viscosity higher the drag force, which opposes the fluid, flow. Again, the transverse magnetic field results a drag force known as Lorentz force, which reduces the momentum boundary layer thickness.

Thus θ_r and M are to decrease horizontal velocity. But on the thermal boundary layer these parameters have opposite effects i.e the thermal boundary layer thickness decreases for the viscosity parameter θ_r and increases for the magnetic parameter M. As can be seen from the graphs that rising of magnetic field intensity results the thermal and concentration boundary layers to expand, resulting in an increase in the wall temperature and concentration gradients and hence increases the heat and mass transfer rates.

Figures 7-8 depict the effects of Darcy parameter Da and stretching parameter β on horizontal velocity $f'(\eta)$ of the fluid. Darcy parameter is directly proportional to permeability. Therefore, as the Darcy parameter increases the resistance due to porous medium decreases and thus momentum boundary layer thickness increases.

But stretching parameter β reduces the momentum boundary layer thickness. Thus, Darcy parameter Da is to increase horizontal velocity of the fluid but stretching parameter β is to decrease horizontal velocity of the fluid in the boundary layer.

Figures 9-11 exhibit the effects of thermal conductivity parameter ε, heat source parameter S and Eckert number Ec on temperature $\theta(\eta)$ of the fluid. It is observed that higher values of ε, S and Ec raise the thermal boundary layer growth which implicates rising in fluid temperature.

This is because of the fact that higher the fluid thermal conductivity higher its ability in conducting heat. Moreover, increasing Eckert number implies increasing store of heat energy in fluid due to frictional heating. The reason for rising temperature in fluid due to heat source is obvious.

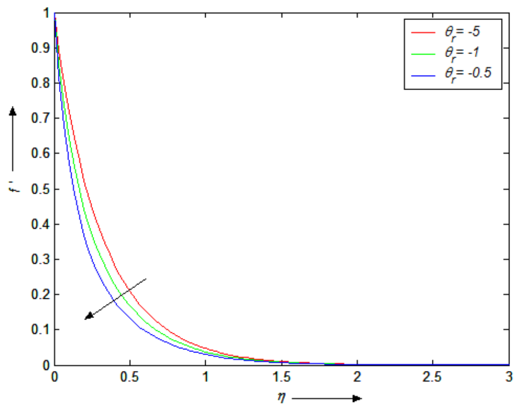

Figure1. Horizontal velocity profiles for various values of θ_r

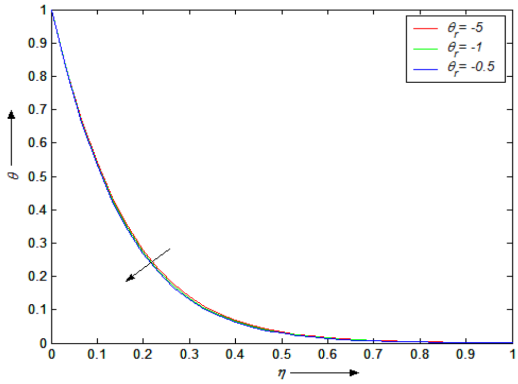

Figure 2. Temperature profiles for various values of θ_r

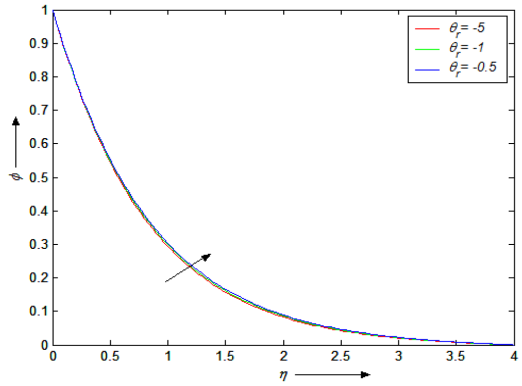

Figure 3. Concentration profiles for various values of θ_r

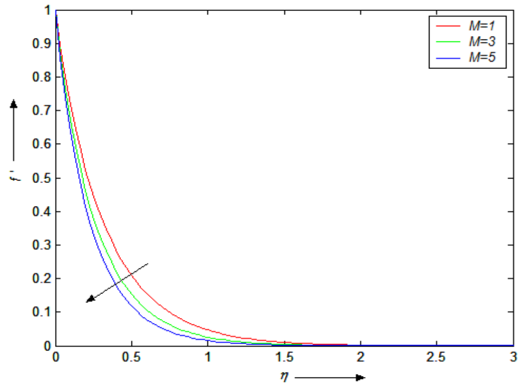

Figure 4. Horizontal velocity profiles for various values M

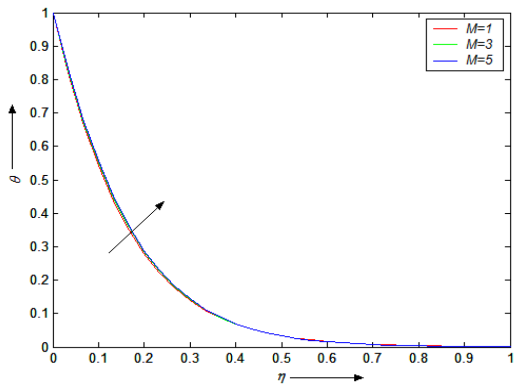

Figure 5. Temperature profiles for various values of M

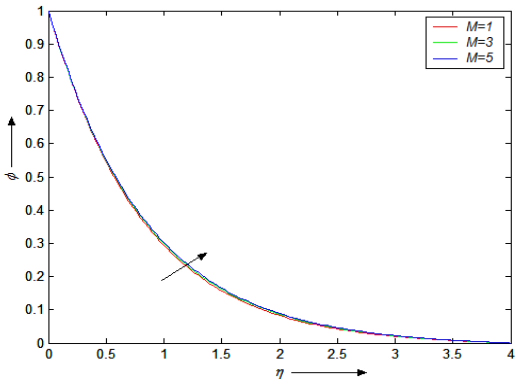

Figure 6. Concentration profiles for various values of M

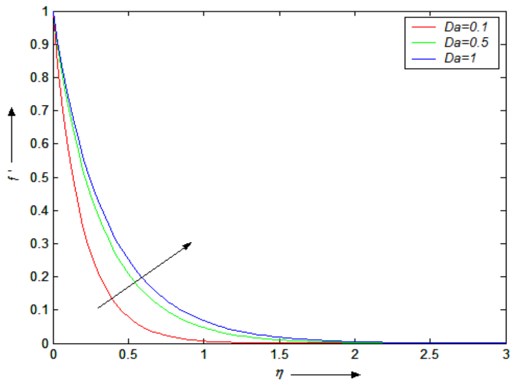

Figure 7. Horizontal velocity profiles for various values of Da

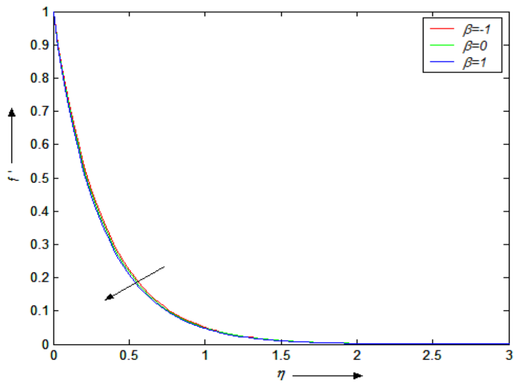

Figure 8. Horizontal velocity profiles for various values of β

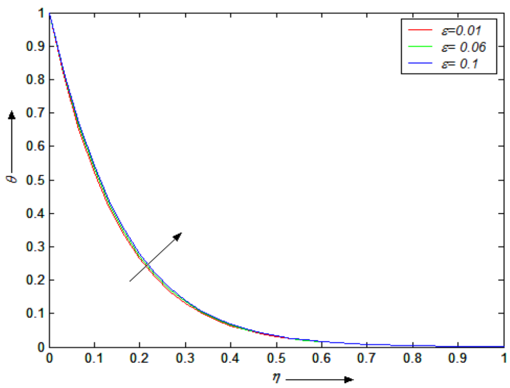

Figure 9. Temperature profiles for various values of ε

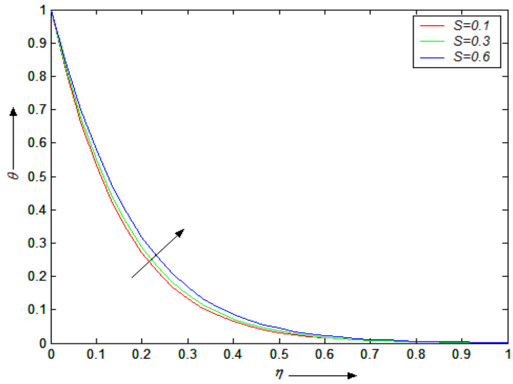

Figure 10. Temperature profiles for various values of S

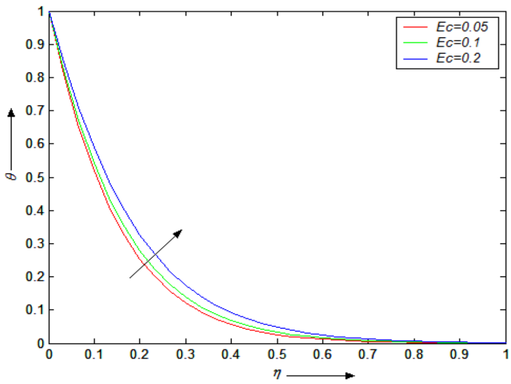

Figure 11. Temperature profiles for various values of Ec

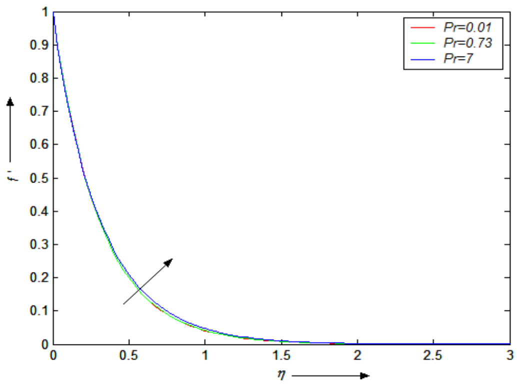

Figure 12. Horizontal velocity profiles for various values of Pr

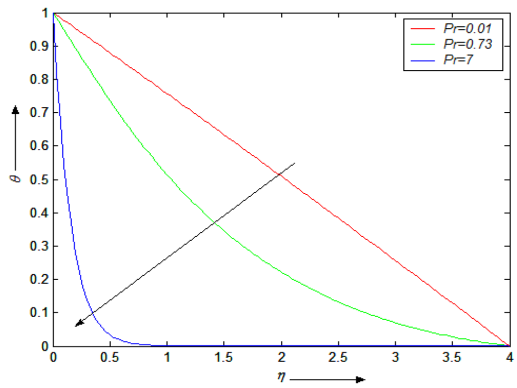

Figure 13. Temperature profiles for various values of Pr

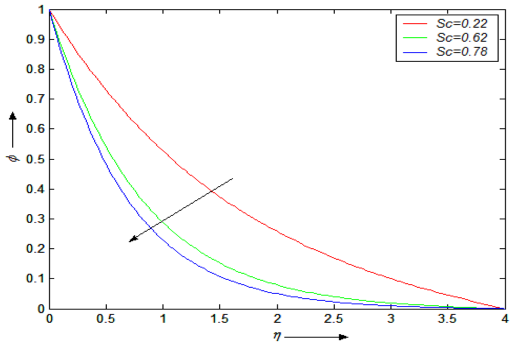

Figure 14. Concentration profiles for various values of Sc

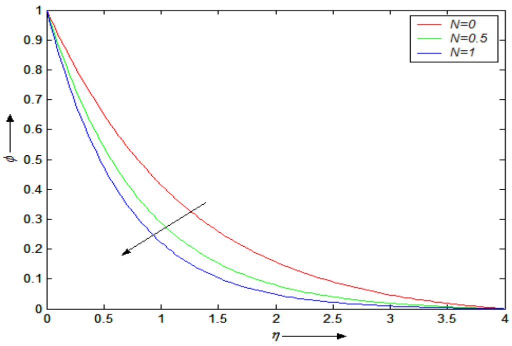

Figure 15. Concentration profiles for various values of N

Figures 12-13 represent the effects of Prandtl number Pr on horizontal velocity $f'(\eta)$ and temperature $\theta(\eta)$ of the fluid. Parametric values considered for Pr are 0.01, 0.73 and 7 corresponding to liquid metal, air and water respectively. It is clear from the figures that Prandtl number Pr enhances the momentum boundary layer thickness but there is a sharp fall of the temperature for high Prandtl number, which leads to thinning of thermal boundary layer. Table1 shows that for higher Pr values wall temperature gradient increases significantly. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Higher Pr value indicates greater momentum diffusivity and lesser thermal diffusivity, resulting in heat being diffused from the surface very slowly and hence temperature of the fluid decreases. As mentioned, Prandtl number Pr governs the relative thickness of momentum and thermal boundary layers. For low Prandtl number, heat diffuses fast in comparison to momentum, which implies the thermal boundary layer in liquid metals is significantly thicker than the momentum boundary layer. Fluids having a lower Prandtl number have higher thermal conductivities, allowing heat to escape the sheet more quickly than fluids with higher Pr values. As a result, Prandtl number may be employed to boost the cooling rate in conducting fluid flows.

The actions of Schmidt number Sc and chemical reaction parameter N on concentration $\phi(\eta)$ of the fluid are shown in the Figures 14-15. Parametric values considered for Sc are 0.22, 0.62 and 0.78 corresponding to Hydrogen, water vapour and ammonia respectively. It is observed that the concentration falls steadily with the increase of distance from the sheet. The drop of concentration becomes more significant for heavier species in presence of chemical reaction. Because higher value of Sc implies higher kinematic viscosity which opposes mass transport phenomena and thus leads to thinning of concentration boundary layer thickness and hence, Sc and N are to decrease species concentration.

To validate the numerical scheme used in the present study a comparison is made in the Table1 to the known numerical results for higher values of the Prandtl number Pr that were obtained by (Gorla and Sidawi[11]) for skin friction $-f''(\eta)$ and wall temperature gradient $-\theta'(\eta)$ when

$n = 1$; $\theta_r \to \infty$; $\beta = 1$; $\varepsilon = 0$;
$Da \to \infty$; $M = 0$; $Re = 0$; $S = 0$;
$Ec = 0$; $Sc = 0$; $N = 0$; $\lambda = 0$.

Table 2 gives the numerical values of skin friction $-f''^{(0)}$, wall temperature gradient $-\theta'^{(0)}$ and wall concentration gradient $-\phi'^{(0)}$when $n = 1$; $Pr = 7$; $Re = 1$; $S = 0.2$; $Ec = 0.1$; $\lambda = 0$. It is observed from the Table 2 that skin friction increases but wall temperature and wall concentration decrease for increasing stretching parameter β, magnetic parameter M, Darcy parameter Da and viscosity parameter θ_r. Wall temperature gradient decreases but skin friction and wall concentration gradient do not alter for thermal conductivity parameter ε. Wall concentration increases for Schmidt number Sc and chemical reaction parameter N but no effects on skin friction and wall temperature gradient are observed for Sc and N.

Table 1. Skin friction $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for various values of Pr

	Previous result (Gorla and Sidawi[11])		Present result	
Pr	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
7	1.014349	1.89046	1.014349	1.890458
10	1.014349	2.30350	1.014349	2.303497
20	1.014349	3.35391	1.014349	3.349943
50	1.014349	5.42474	1.014349	5.424762
70	1.014349	6.46221	1.014349	6.458784

Table 2. Skin friction $-f''(0)$, wall temperature gradient $-\theta'(0)$ and wall concentration gradient $-\phi'(0)$

β	M	ε	Da	θ_r	Sc	N	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	1	0.1	1	-5	0.62	1	1.8963	0.6891	0.8942
0	1	0.1	1	-5	0.62	1	1.6764	0.7834	0.8981
1	0	0.1	1	-5	0.62	1	1.5481	0.9072	0.9045
1	1	0	1	-5	0.62	1	1.8963	0.7383	0.8942
1	1	0.1	0	-5	0.62	1	1.5916	0.8899	0.9036
1	1	0.1	1	∞	0.62	1	1.7323	0.7153	0.8978
1	1	0.1	1	-5	1	1	1.8963	0.6891	1.1402
1	1	0.1	1	-5	0.62	0	1.8963	0.6891	0.4599

Conclusion

From the above discussions, following conclusions can be made:

(i). Viscosity parameter θ_r and magnetic parameter M reduce fluid horizontal velocity but enhance species concentration. However, fluid temperature decreases for the viscosity parameter and increases for the magnetic parameter. Skin friction increases by 9.46% and wall temperature and wall concentration gradients decrease by 3.66% and 0.40% respectively with the increase in viscosity parameter from $\theta_r \to \infty$ (non-viscous) to θ_r =-5. Also, with the rising value of magnetic parameter M from 0 to 1, skin friction increases by 22% whereas wall temperature and wall concentration gradients decrease by 24% and 1.13% respectively.

(i). Darcy parameter Da enhances horizontal fluid velocity while stretching parameter β has opposite effects. Skin friction increases by 19% and 13% with the increase in values of Da and stretching parameter β from 0 to 1 respectively. But wall temperature gradients are decreased by 22% and 12% while wall concentration gradients are decreased by 1.04% and 0.43% respectively for changing value of the Darcy parameter Da and the stretching parameter β from 0 to 1.

(ii). Thermal conductivity parameter ε, heat source parameter S and Eckert number Ec enhance fluid temperature. Wall temperature gradient decreases by 6.66% due to the rising value of thermal conductivity parameter from 0 to 0.1. There are no changes in skin friction and wall concentration gradients for the thermal conductivity parameter ε.

(iii). Prandtl number Pr is to increase horizontal velocity of the fluid but to decrease fluid temperature. Prandtl number has no significant effects on species concentration within the boundary layer.

(iv). Schmidt number Sc and chemical reaction parameter N are to decrease the species concentration of the fluid. Both the parameters Sc and N do not have any impact on the skin friction and on the wall temperature gradient. But wall concentration gradient increases by 27.5% and 94.4% when Sc increases from 0.62 to 1 and N increases from 0 to 1 respectively. Thus mass transfer takes place within the boundary layer.

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Nomenclature

a	stretching rate
b,c	constants
$B(x)$	magnetic field strength
B_0	magnetic field constant
C	fluid concentration
c_p	specific heat at constant pressure
C_w	concentration of the fluid on the sheet
C_∞	concentration far away from the sheet
D	Mass diffusivity
Da	Darcy parameter
Ec	Eckert number or viscous dissipation
F	dimensionless velocity variable
K	thermal conductivity
k_c	coefficient of chemical reaction
k_p	permeability
k_∞	thermal conductivity away from the sheet
M	magnetic parameter
m_w	surface mass flux
N	exponent
N	chemical reaction parameter
Pr	Prandtl number
Q	coefficient of heat source
q_w	surface heat flux
Re	Reynolds number
S	heat source or heat generation parameter
Sc	Schmidt number
T	fluid temperature
T_r	transformed reference temperature
T_w	temperature on the sheet
T_∞	temperature away from the sheet
u	velocity in x-deirection
u_w	velocity at the sheet
v	velocity in y-direction
v_w	suction/injection velocity at the sheet
x	horizontal distance
y	vertical distance
ε	thermal conductivity parameter
δ	thermal property of the fluid
τ_w	surface shear stress
λ	suction/injection parameter
η	similarity variables
μ	dynamic viscosity
μ_w	viscosity at the wall
μ_∞	constant dynamic viscosity
σ	electrical conductivity
ν_∞	constant kinematic viscosity
β	stretching parameter
ψ	stream function
ρ_∞	constant fluid density
θ	dimensionless temperature variable
θ_r	dimensionless viscosity parameter
ϕ	dimensionless concentration parameter

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