# Monte Carlo Method for Calculating View Factor of Truncated Cone Radiators 

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#### Abstract

To address radiative heat transfer problems, the determination of view factors is crucial. In this study, the focus is placed on the calculation of the view factor using the Monte Carlo method, specifically for truncated cone radiators. Although reference books offer theoretical relations for computing the view factor, a new approach employing the Monte Carlo method is utilized to ensure the accuracy of the general solution. To measure the accuracy, three types of cases are considered: positive, negative, and zero-angle truncated cones with a fixed disk (ring) at the base of the cone. The results are presented for various ratios between the height of the truncated cone and the radii of the ring and base side of the cone. Additionally, the impact of different angles of the truncated cone on the view factor is investigated. In the zeroangle case, five different $\mathrm{L} / \mathrm{r}_{1}$ are examined, in the positive angle case, seven different positive angles in two different $\mathrm{L} / \mathrm{r}_{1}$ are studied, and in the negative angle case, three negative angles in three different $L / r_{1}$ are studied. For positive angles, the maximum difference between the results of Monte Carlo method and theoretical method is $42.81 \%$ and occurred in $\mathrm{L} / \mathrm{r}_{1}$ equal to 5 and 40 degrees. While for zero-angle the maximum difference is $30.16 \%$ and occurred in $\mathrm{L} / \mathrm{r}_{1}$ equal to 10 . In the negative angle case, the maximum difference is $36.66 \%$ and occurred in $L / r_{1}$ equal to 0.2 and -15 degrees.


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## 1. Introduction

Radiation heat transfer is an important energy transfer method at high temperatures. The relative orientation of surfaces influences radiation heat transfer. The view factor is a parameter used to represent orientation's effects on radiation heat transfer between two surfaces. Regardless of surface temperature or geometry, this parameter is an independent quantity. When complex geometries are involved and surfaces and shapes are arranged arbitrarily, the view factors for the particular geometry and arrangement of surfaces need to be computed manually. Appropriate techniques must be used, employing numerical algorithms and computers
[1]. The Monte Carlo method is a commonly used numerical solution that is efficient and easy to implement. As far back as Howell and Perlmutter, Monte Carlo methods have been used to study radiation heat transfer [2]. Using Monte Carlo methods, one can simulate physical processes using analogous models or the statistical characteristics of physical processes. By performing statistical sampling experiments on a computer, it is possible to approximate solutions to various mathematical problems. The technique has been widely used in different fields, ranging from economics to nuclear physics and even the arrangement of wind turbines in a wind farm. Investigation of past examinations shows that

[^0]Monte Carlo offers an important technique for anticipating the advantages of setup factors as it can consolidate all basic impacts in a radiative heat transfer simulation without guessing [3].

Nonetheless, the Monte Carlo method has some disadvantages. One is the requirement for computer time; the other is the inherent statistical variance of the results. For most problems where understanding the radiation field is desirable, the method is fairly efficient at simulating complex problems. Nevertheless, if only radiative intensity within a small area of solid angles is required, the method may prove ineffective [4]. Maltby and Burns [5] researched execution, intermingling, and precision in a three-layered Monte Carlo radiative heat transfer simulation with a code as well as the abilities to blend, grouped spectral material properties, diffuse and specular reflection models, transmission through surfaces, and simulation of beam radiation. Miyahara and Kobayashi have additionally developed another mathematical strategy for obtaining the view factor for an axially symmetrical geometry [6]. Comparing this method to the area integration and Monte Carlo methods, it was observed that it was 19 times and three times faster, respectively. In a study by Quaky et al. [7], a two-dimensional Monte Carlo model of an industrial furnace's interior was applied to a classic radiant energy exchange problem. The heat sink was a parallel row of infinitely long tubes connected to a source as an infinite radiating plane. Two years later, Hong and Welty developed this method for radiation heat transfer in a three-dimensional enclosure containing a horizontal circular cylinder [8]. A fast Monte Carlo scheme was introduced in the examination by Mazumder and Kresch [9]. The essential calculation was the classical ray tracing algorithm. Moreover, a modified form of the binary spatial partitioning (BSP) algorithm was implemented to accelerate ray tracing by at least a factor of 3. In a study done by Xia et al. [10], discretizing the medium into many sub-layers and utilizing a linear refractive index approximation for each sub-layer, a curve Monte Carlo method was created to tackle the radiative heat transfer in an absorbing and scattering gradient-index medium. In honeycomb-type transparent insulation materials, Schweiger et al. [11] applied this method for both conduction and radiation heat transfer. Mirhosseini and Saboonchi [12] utilized the Monte Carlo strategy to decide a plate's view factor, including strip elements to a circular cylinder. Additionally, Mirhosseini and Saboonchi [13] dedicated two parallel circular cylinders as a case in heating and cooling processes (e.g., the transfer table in hot rolling) for determining the plate's configuration factor. Wei and Jiang [14] determined the view
factors between internal heat source surfaces and their surroundings to better understand thermal radiation in rooms. They compared the results with numerical results obtained by the Monte Carlo method. View factors were calculated by Walker et al. [15] for an operational fiber drawing furnace using both numerical integration and the Monte Carlo method. Ravishankar et al. [16] presented algorithms to apply the modified differential approximation (MDA) to arbitrary geometry, in particular, geometry with obstructions and inhomogeneous media, to remove the shortcomings of the firstorder spherical harmonics method (or P1 approximation). The general procedure was validated for both two-dimensional (2D) and three-dimensional (3D) geometries against benchmark Monte Carlo results. A study investigated the effects of boundary conditions and microstructural parameters by Monte Carlo simulation of radiative heat through fibrous media [17]. Mazumder and Ravishankar [18] used a general formulation accompanying numerical procedures to calculate diffuse view factors between arbitrary planar polygons. The results were compared to exact analytical solutions (when available) or Monte Carlo results. They argued that the errors in the Monte Carlo results increase when the two surfaces are far from each other. Matthew et al. [19] improved an approach realized using the computer program REFORM based on the well-known Monte Carlo algorithms. The method allowed determining the view factors of radiative heat exchange area in large-scale furnaces wherever the method of direct numerical integration is difficult to use due to their geometries. Wang [20] developed a stochastic algorithm to estimate view factors between canyon facets in the presence of shade trees based on Monte Carlo simulation, where an analytical formulation cannot be used in complex geometry. Three methods have been proposed for calculating the view factor of a strip relative to a parallel semicylinder in Hajji et al. [21]. There were significant differences between the results obtained by the Monte Carlo method and the analytical solution. In the study by Liu et al. [22], a model based on the Monte Carlo ray-tracing method was built to analyze the radiative energy loss in the polysilicon CVD reactor. The effects of the rods number, thermal shield, and surface emissivity on the radiative heat loss of a CVD reactor were investigated in detail. Frank et al. [23] modeled the heat flows in a switch cabinet by calculating the surface-to-surface radiation exchange using a Monte Carlo method and a quasi-Monte Carlo method. Some other related publications can be found in [24, 25].

The view factor calculation in three dimensional complex geometries is only examined through individual theoretical solutions according the cases. In literature, these solutions are available for a few problems with specific applications, while the well-known theoretical methods such as Cross String method that can not be used excepting for calculating view factors in two dimensional problems. Furthermore, the Monte Carlo method has shown superior and more accurate results for calculating view factors in previous studies in different cases due to its statistical basis. In some cases, a large difference between the results of theoretical solutions and the Monte Carlo method has been observed, such as a study which Mirhosseini, as the author of the present article, has seen previously with his colleagues [21]. Importantly, none of previous studies have rigorously compared the results of theoretical solutions and the Monte Carlo method for a threedimensional case, encouraging the authors to choose. Therefore, the use of the Monte Carlo method is adopted here for calculating view factor of truncated cone radiators, due to its ability to deal with three-dimensional complexities that cannot be considered by the available solution method. This allows examination of the above-mentioned issue and enables direct comparison of the Monte Carlo and theoretical methods in a three-dimensional context, which is absent in previous researches.

Truncated cone, characterized by its cone shape with a portion cut off at the top, serve diverse applications across different engineering fields and scientific domains. For example, in antenna systems, they contribute to controlling wave patterns. Additionally, these shapes find use in heat exchangers for optimizing radiative heat transfer. Therefore, the various applications of truncated cone geometry are driven by its ability to influence and enhance specific characteristics within a given field.

In the present study, the configuration factor of the truncated cone radiator is calculated using the Monte Carlo method and compared to results from the theoretical solution available in the literature. The evaluation is conducted for three cases: positive, negative, and zero angle truncated cones with a fixed disk (ring) at the base of the cone. The results are analyzed for various ratios between the height of the truncated cone and the radii of the ring and base side of the cone. Additionally, the impact of differing truncated cone angles on the view factor is investigated. Through this comparative analysis and examination of the effects of varying cone angles, the aim is to enhance the understanding of the view factor calculation for truncated cone radiators. The Monte Carlo
method is implemented because it ensures accuracy for the solution, generally. By conducting this kind of studies, the reliability of various solutions for radiative heat transfer problems involving complex geometries in practical applications, is determined.

## 2. Methods

Geometric relationships are needed to determine how the faces view each other in order to calculate radiative heat transfer. This mathematical correlation presents a boundary identified as the shape factor. Radiant energy leaving a surface incident upon a reference surface is determined by the view factor. Based on the solid angle subtended by one surface, they are dependent on problem geometry.


Fig. 1. Geometric parameters between two differential areas
Fig. 1 shows the view factor between two deferential areas, which is calculated by [26]:

$$
\begin{equation*}
d F_{d 1-d 2}=\frac{\cos \left(\theta_{1}\right) \cdot \cos \left(\theta_{2}\right)}{\pi \cdot S^{2}} d A_{2} \tag{1}
\end{equation*}
$$

Also, explicitly, the view factor, F, between a differential region and surface is expressed as:

$$
\begin{equation*}
F_{d 1-2}=\int_{A_{2}} \frac{\cos \left(\theta_{1}\right) \cdot \cos \left(\theta_{2}\right)}{\pi \cdot S^{2}} d A_{2} \tag{2}
\end{equation*}
$$

View (configuration, shape, and exchange) factors are dependent upon the shape and orientation of the surfaces and their distance from each other [26]. Implementing relations (1) and (2) can be written as:

$$
\begin{equation*}
F_{d 1-2}=\int_{A_{2}} d F_{d 1-d 2} \tag{3}
\end{equation*}
$$

Similarly, the configuration factor of two ordinary surfaces is calculated by:

$$
\begin{equation*}
F_{12}=\frac{1}{A_{1}} \int_{A_{2}} \int_{A_{1}} \frac{\cos \left(\theta_{1}\right) \cdot \cos \left(\theta_{2}\right)}{\pi \cdot S^{2}} d A_{1} d A_{2} \tag{4}
\end{equation*}
$$

Also, the reciprocity relationship for configuration factor between these areas is obtained as:

$$
\begin{equation*}
A_{2} F_{2-1}=A_{1} F_{1-2} \tag{5}
\end{equation*}
$$

According to relation (4), the calculation of shape factor between two no infinitesimal areas requires the solution of double area integral or fourth-order integration. Even for many simple geometries, computing such integrals has many efforts. So, for determining the view factor, other methods must be used.

Several other methods exist for calculating shape factors that are introduced in brief. Some methods that are called "special methods" compute view factors indirectly. These methods calculate shape factors with geometrical restrictions and are just implemented for exceptional geometries. "Crossed strings method," "unit sphere method," and "inside sphere method" are some of them. Another method with no limitation is the statistical method like the "Monte Carlo method."

### 2.1. Monte Carlo method

The Monte Carlo method is based on a statistical approach to calculating shape factors. Probability is the driving force behind this method (i.e., chance). As a consequence of the Monte Carlo method, all of the energy emitted by a given area is replaced with the sum of $N$ rays. These rays are equivalent in energy. Radiation coming from an element surface can come from a variety of sources, for example, randomly [26] or all coming from the center of the face of the element. Some rays will reach another surface, while others will not. The view factor, which links finite elements i and j , can be expressed as follows:

$$
\begin{equation*}
F_{i-j}=\frac{m}{N} \tag{6}
\end{equation*}
$$

where N is the total number of rays emitted from surface $i$ and $m$ is the number of rays hitting surface j . The complementary descriptions exist in the reference [3].

This research investigates the view factor in a truncated cone radiator by theoretical and Monte Carlo methods. Monte Carlo algorithm is implemented for three cases; positive, negative, and zero angle truncated cone with a fixed disk (ring) at the base of the cone. For this purpose, a program was written in MATLAB software. Results are presented for different ratios between the height of the truncated cone and the radii of the ring and base side of the cone. The effect of different angles of the truncated cone is also investigated on the view factor. This method aimed to calculate the view factor by using the geometric parameters of the truncated cone radiator shown in Figure 2.


Fig. 2. Schematic view of the truncated cone and disk with geometric parameters

There is no specific rule to estimate the number of rays needed to solve a Monte Carlo problem. In the present study, for each case, the ray number is taken to 1010 to reach a stabilized result. Naraghi and Chung have offered the analytical solution in [27], as seen in the relation (7). A recent reference has represented the relation for calculating the view factor of an annular disk (ring) to the truncated cone. But in this research, the focus is on estimating the view factor of the truncated cone to the ring. Therefore, the reciprocity relation has been used to present the theoretical view factors.

$$
\begin{align*}
H= & \frac{h}{r_{2}} \quad R=\frac{r_{1}}{r_{2}} \\
A= & {\left[H^{2}+(1+R+H \tan \alpha)^{2}\right]^{\frac{1}{2}} } \\
A= & {\left[H^{2}+(1-R-H \tan \alpha)^{2}\right]^{\frac{1}{2}} } \\
C= & (1-R)^{\frac{1}{2}} \quad D=(1+R)^{\frac{1}{2}} \quad E=\cos ^{2} \alpha\left(1-R^{2}\right) \\
F_{1-2}= & \frac{1}{\pi\left(1-R^{2}\right)}\left\{-A B \tan ^{-1} \frac{A C}{B D}+(C D)^{2} \tan ^{-1} \frac{D}{C}\right.  \tag{7}\\
& +\frac{\sin \alpha}{\cos ^{2} \alpha}\left[\left(H^{2}+\frac{2 H R}{\tan \alpha}\right) \tan ^{-1} \frac{E^{\frac{1}{2}}}{H}+E \tan ^{-1} \frac{H}{E^{\frac{1}{2}}}\right] \\
& \left.+\left(\frac{H^{2}}{2 \cos ^{2} \alpha}+H R \tan \alpha\right) \cos ^{-1} R\right\}
\end{align*}
$$

In order to better understand the physics involved, a truncated cone with a disk attached to the downward side of the cone is shown in Fig. 3.


Fig. 3. Situation of tangent oblique line on truncated cone, variation range of $\theta$ angle and ' $x$ '

The result is that any oblique line tangent to the cone can only see a portion of its front area in the case of an oblique line tangent to the cone; therefore, the calculations do not have to cover the entire lateral extent of the cone. In the figure, the imaginary point (1) is on an oblique line tangential to the cone, and each end can only see the front part of the ring area on a diagonal line. The points (2) and (3) are on the ring plane horizontally. The angle of $\theta$ can vary between 0 and 180 degrees. Also, the length of 'x' changes based on $\theta$.


Fig. 4. Required parameters definition for Monte Carlo method calculation

In Fig. 4, the oblique line in the vertical plane, the angle of $\theta$, and the length of " $x$ " in the horizontal plane can demonstrate the simple geometric parameters of the problem. In the following, the angle of the cone is shown by $\beta$.

Equations about the limitation of striking the disk can be derived by dividing the height of the truncated cone by some point elements. It helps simplify writing mathematically relationships by choosing different angles and locations on the oblique tangent line on the truncated cone and by choosing different positions on the disc's surface and the portion of the disk where energy can strike. When $\theta=0$, the angle between the oblique tangent line on the truncated cone and the ' x ' length equals $90^{\circ}$. By increasing the angle of $\theta$ leading to $180^{\circ}$, the mentioned angle changes permanently. Thus, these changes can be related mathematically. Using these relations, the angle between the oblique line and the line that creates the arch of $\theta$ can be calculated. By imagining ' $\mathrm{OA}^{\prime}$ on the joint line between two perpendicular planes, $x^{\prime}$ will be obtained. A perpendicular line is drawn from the point obtained by imaging to cut the extension of ' $O B$ ' in a point. Then a triangle is created on the horizontal plane. Also, the amount of " $k$ " can be calculated in two ways. It can be estimated between the side ' $O A$ ' and the side ' $O B$ ' portion. The angle of $\psi$ is between 'OA' and 'OB.' According to the below relations, $\psi$ can be calculated from the cosine law. By having the
angle of $\psi$ and also writing the sine law in the triangle 'OAB,' the final relations can be obtained:

$$
\begin{align*}
& x^{\prime}=L^{\prime} \tan (\beta) \\
& \tan (90-\theta)=\left(\frac{p}{L^{\prime} \tan (\beta)}\right) \\
& \cos (90-\theta)=\frac{L^{\prime} \tan (\beta)}{\sqrt{p^{2}+\left(L^{\prime} \tan (\beta)\right)^{2}}} \\
& k=\sqrt{p^{2}+L^{\prime 2}} \\
& \cos (\psi)=\frac{k^{2}-\left(\frac{L^{\prime}}{\cos (\beta)}\right)^{2}-\left(\frac{L^{\prime} \tan (\beta)}{\cos (90-\theta)}\right)^{2}}{-2\left(\frac{L^{\prime}}{\cos (\beta)}\right)\left(\frac{L^{\prime} \tan (\beta)}{\cos (90-\theta)}\right)}  \tag{8}\\
& \frac{x}{\sin (\varphi)}=\frac{\sqrt{x^{2}+\left(\frac{L^{\prime}}{\cos (\beta)}\right)^{2}-2 x\left(\frac{L^{\prime}}{\cos (\beta)}\right) \cos (\psi)}}{\sin (\psi)} \\
& 0 \leq \tan (\text { Random angle }) \leq \tan (\varphi)
\end{align*}
$$

Thus, the angle of $\varphi$ (angle between 'OA' and ' AB ') will be calculated. Every one of the rays emitted randomly can satisfy the above condition. One unit is added to a counter in the original program code. Indeed, the counter is used directly for calculating the view factor. One application of this geometrical configuration can be for very hot truncated cone tubes, while the hot fluid flows internally. Whereas the tube's temperature is very high, the application of rings is important. It should be noted that for $\theta=0 \circ$ and $\theta=180^{\circ}$, the amount of ' $x$ ' can be calculated from $\sqrt{r_{2}{ }^{2}-r_{1}^{2}}$. Also, for $\theta=90^{\circ}$, ' x ' is equal to ( $\mathrm{r}_{2}-\mathrm{r}_{1}$ ). Relation between $\theta$ and ' $x$ ' can easily be obtained.

## 3. Results and discussion

To assess the capability of these methods, three kinds of cases are considered; positive, negative, and truncated cone with a fixed disk (ring) at the base of the cone. For comparison, the figures show the theoretical and Monte Carlo method results. Generally, the configuration factor decreases by increasing the cone length ratio to the inner radius of the disk (radius of the base side of the cone). In all cases, the shape factor increases as the outer radius is increased relative to the inner radius of the disk. The results can show which method is more reasonable and accurate rather than the other one. In the following sub-sections, these three cases are presented.

### 3.1. Configuration factor for zero angle

The configuration factor of the zero-angle truncated cone to disk will be considered in this part. As shown in Fig. 5, a truncated cone is converted into a circular cylinder. Thus, the determination of the view factor of a cylinder to a coaxial disk will be discussed.

In Fig. 6, in each constant value $L / r_{1}$, by increasing the ratio of the outer radius to the inner radius of the disk, the shape factor increases. Also, with increasing the ratio $L / r_{1}$ at the constant ratio $r_{2} / r_{1}$, the configuration factor decreases. This behavior has physical justification; when the length of the cylinder increases, the cylinder can see a more extensive area, and, in this manner, a fraction of incident rays to total emitted rays will reduce. The results exhibit that the view factors obtained by the Monte Carlo method are higher than the theoretical solution since the cone angle is zero.

### 3.2. Configuration factor for the positive angle

In this part of the present work, the shape factor of the positive angle truncated cone radiator is discussed. As shown in Fig. 7, when the angle is positive, without changing the other geometric parameters, the lateral area of the cone is increased. Evaluations by considering two ratios $L / r_{1}$ were performed in Fig. 8 and 9. With increasing the value $r_{2} / r_{1}$ corresponding to all angles, the shape factors are increased, although the convexity and concavity of the curves change. In each figure, by increasing the angle at a constant ratio of $r_{2} / r_{1}$ and $L / r_{1}$, the shape factors increase to a maximum value in an individual angle and then decrease. It can be termed an optimized cone angle which depends on the geometric parameters. This fact occurs because larger than a particular positive angle, the number of incident rays (to the disk) divided into total emitted rays (from the cylinder) decreases. Also, it can be said that by comparing these figures at the same angle and the same ratio $r_{2} / r_{1}$, with increasing the ratio $L / r_{1}$, the configuration factors decreased. In Fig. 8 for $r_{2} / r_{1}=4.5$, the view factor calculated by the
theoretical method at $80^{\circ}$ is higher than the value at $0^{\circ}$ cone angle. In contrast, in the Monte Carlo solution, this comparison is vice versa. In the theoretical solution, the view factor corresponding to $60^{\circ}$ overtakes from $40^{\circ}$ at $r_{2} / r_{1}=3.5$, while it takes place in $r_{2} / r_{1}>4$ for the Monte Carlo solution. In Fig. 9, similar behavior can be observed for different angles and ratios. Therefore, the best angle to get the highest view factor is a function of $L / r_{1}, r_{2} / r_{1}$ and calculating method.

### 3.3. Configuration factor for negative angle

As shown in Fig. 10, when the angle is negative, the cone's lateral area is decreased without changing the other geometric parameters. Before discussing the results of this section, it is necessary to say that one restriction must be considered for selecting negative angles. The chosen angle should not be smaller than $\tan ^{-1}\left(r_{1} / L\right)$, because at the mentioned angle, the truncated cone is changed to a cone, and at a smaller angle, two crossed cones will be generated. Figures 11 to 13 illustrate that by increasing the ratio of $r_{2} / r_{1}$ for all negative angles, the configuration factors are improved. Also, when the absolute value of the negative


Fig. 5. Schematic view of zero angle truncated cone


Fig. 6. a) Theoretical and b) Monte Carlo solution for zero angle


Fig. 7. Schematic view of positive angle truncated cone


Fig. 8. a) Theoretical and b) Monte Carlo solution for positive angle ( $L / r_{1}=3$ )


Fig. 9. a) Theoretical and b) Monte Carlo solution for positive angle ( $L / r_{1}=5$ )


Fig. 10. Schematic view of negative angle truncated cone


Fig. 11. a) Theoretical and b) Monte Carlo solution for negative angle ( $L / r_{1}=0.2$ )


Fig. 12. a) Theoretical and b) Monte Carlo solution for negative angle ( $L / r_{1}=3$ )


Fig. 13. a) Theoretical and b) Monte Carlo solution for negative angle ( $L / r_{1}=5$ )
angle is increased at the constant ratio $r_{2} / r_{1}$, the shape factors will reasonably decrease by using the Monte Carlo method. In contrast, they will surprisingly increase by using the theoretical solution. The theoretical relation cannot calculate the view factors for the negative cone angle with a rule of thumb. In fact, at more negative angles, the angle between the truncated cone and disk is
more obtuse. Thus, the number of incident rays (to the disk) divided into the total beams (from the cone) decreases. Also, by comparing these figures at the same angle and the same ratio $r_{2} / r_{1}$ , the higher proportion of $L / r_{1}$ the configuration factor becomes lower. Also, the Monte Carlo results are less sensitive to the cone angle in a higher ratio.

## 4. Conclusions

The present study utilized the Monte Carlo method, in addition to the theoretical solution, for calculating the view factor in the practical case of truncated cone radiators. The analysis was conducted for three configurations: positive, negative, and zero-angle truncated cones with a fixed disk (ring) at the base of the cone. The results were examined for various ratios between the height of the truncated cone and the radii of the ring and base of the cone. Additionally, the impact of different angles of the truncated cone on the view factor is investigated. In the zeroangle case, five different $\mathrm{L} / \mathrm{r}_{1}$ are examined, in the positive angle case, seven different positive angles in two different $\mathrm{L} / \mathrm{r}_{1}$ are studied, and in the negative angle case, three negative angles in three different $\mathrm{L} / \mathrm{r}_{1}$ are studied. The findings show that for positive angles, the maximum difference between the presented results of Monte Carlo method and theoretical method is $42.81 \%$ that occurred in $\mathrm{L} / \mathrm{r}_{1}$ equal to 5 and the cone angle of 40 degrees, whereas for zero-angle, the maximum difference is $30.16 \%$ and occurred in $\mathrm{L} / \mathrm{r}_{1}$ equal to 10 . In the negative angle case, the maximum difference is $36.66 \%$ that occurred in $\mathrm{L} / \mathrm{r}_{1}$ equal to 0.2 and the cone angle of -15 degrees. Such a large difference between the results of theoretical solution and Monte Carlo method has been observed in other cases previously. However, the results showed that the overall trends match between the two methods. This was seen in Figures 6, 8, 9, 11, 12, and 13. The aligned trends mean both methods provide basically the same understanding of what is being calculated. The main reason of lower accuracy of the theoretical solution is that it relies on simplifying assumptions that break down at complex geometries. Specifically, the theoretical method utilizes approximations to model the radiative heat transfer that become less accurate as the cone angle decreases. This includes assumptions about the uniformity of radiative intensity over the cone surface and modeling the cone as a differential surface element. At very small cone angles, these approximations lead to larger errors. In contrast, the Monte Carlo method directly simulates the probabilistic behavior of individual rays without any simplifications. This allows the Monte Carlo to better deal with the sharp edges and nearly flat surfaces of cone angles.

## Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article. In addition, the authors have entirely observed the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or
falsification, double publication and/or submission, and redundancy.

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