Journal of Heat and Mass Transfer Research 10 (2023), Serial Number 20, 329 – 340

Semnan University

Journal of Heat and Mass Transfer Research

Journal homepage: https://jhmtr.semnan.ac.ir

ISSN: 2383-3068

*Research Article*

# Thermal Performance of Convective-Radiative Transfer Longitudinal Moving Rod with Variable Thermal Conductivity

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**ARTICLE INFO**

***Article history:***

Received: 2023-02-04

Revised: 2024-01-13

Accepted: 2024-01-13

***Keywords:***

Moving convective-radiative rod;

Temperature-dependent thermal conductivity;

Least square method;

Galerkin method.

**ABSTRACT**

An analysis has been performed to study the problem of the thermal performance of a continuously moving convective-radiative rod with variable thermal conductivity. Highly accurate semi-analytical methods called the least Square method (LSM) and the Galerkin method (GM) are introduced and then used to obtain a nonlinear temperature distribution equation in a fin that allows for more accurate measurements that could make the investigation stand out. This research investigated the influence of various parameters on heat transfer in a continuously moving convective-radiative rod. The parameters examined include the convective-conductive factor (*Ncc*), dimensionless thermal conductivity coefficients (*a*), radiative-conductive parameter (*Nrc*), Peclet number (Pe), dimensionless convective ($\theta c$), and radiative sink temperatures ($\theta r$). An increase in the dimensionless thermal conductivity coefficient (a) led to higher dimensionless temperatures within the rod, indicating an amplification of conductive heat transfer. The convective-conductive parameter (*Ncc*) demonstrated a direct relationship with heat loss. In contrast, the radiative-conductive parameter (*Nrc*) exhibited an inverse relationship between radiative heat transfer and local temperature within the fin. A rise in the Peclet number was associated with higher dimensionless temperatures, indicating a faster-moving rod. Additionally, variations in dimensionless convective and radiative sink temperatures affected temperature profiles, with higher sink temperatures resulting in increased dimensionless temperatures. Notably, the dimensionless radiative sink temperature was found to have a more significant impact on overall dimensionless temperature than the convective sink temperature. These findings underscore the intricate interplay of factors governing heat transfer and temperature distribution in the moving rod system. The importance of this work lies in its comprehensive analysis of the intricate interplay of parameters affecting heat transfer and temperature distribution in continuously moving convective-radiative rods, providing valuable insights for optimizing industrial processes and engineering applications.



## 1. Introduction

Fins are frequently used in many heat transfer applications to improve performance. On the other hand, for many years, a high heat transfer rate with reduced size and cost of fins have been primary targets for several engineering applications such as heat exchangers, economizers, superheaters, conventional furnaces, gas turbines, etc. [1,2]. Some engineering applications , such as airplanes and

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**Cite this article as:**
Hoshyar, H. A., Johari, M., Domiri Ganji, D., 2023. Thermal Performance of Convective-Radiative Transfer Longitudinal Moving Rod with Variable Thermal Conductivity. *Journal of Heat and Mass Transfer Research*, 10(2), pp. 329-340.
https://doi.org/10.22075/JHMTR.2024.29819.1416

motorcycles, also require lighter fins with higher rates of heat transfer. Increasing the heat transfer depends on the heat transfer coefficient (h), the surface area available, and the temperature difference between the surface and the surrounding fluid. Nonlinear problems and phenomena play an important role in applied mathematics, physics, engineering, and other branches of science, especially some heat transfer equations. Except for a limited number of these problems, most of them do not have precise analytical solutions. Therefore, these nonlinear equations should be solved using approximation methods. Perturbation techniques are too strongly dependent upon the so-called "small parameters" [3]. Other different methods have been introduced to solve a nonlinear equation such as the δ-expansion method [4], Adomian's decomposition method [5], Homotopy Perturbation Method (HPM) [6–8], and Variational Iteration Method (VIM) [9–11], Homotopy analysis method [12-14] and Galerkin Method [15], which has been successfully applied to solve many types of the nonlinear problem.

Various studies have been conducted on thermal distribution through an annular and longitudinal fin. Varun Kumar et al. [16] examined temperature distribution in a conductive-radiative annular fin with power law-dependent thermal properties using the differential transform method (DTM)-Pade approximant. Results showed temperature distribution increases with heat generation but decreases with higher thermogeometric and radiative-conductive parameters. Hoshyar et al. [17] examined thermal behavior in a porous fin with temperature-dependent internal heat generation using the collocation method (CM) and the homotopy perturbation method (HPM). Varun Kumar et al. [18] investigated how heat spreads in a moving plate using a non-Fourier heat flux model. They found that higher convection-conduction and radiation-conduction parameters reduce heat dispersion, while a higher Peclet number increases it. Sowmya et al. [19] examined how a magnetic field affects the thermal performance of a rectangular-profiled longitudinal fin. The results showed that higher thermal conductivity improves the fin's temperature profile, while the Hartmann number and thermogeometric parameter have the opposite effect. Jayaprakash et al. [20] focused on the thermal distribution through a moving longitudinal trapezoidal fin with variable temperature-dependent thermal properties using the Differential Transform Method (DTM) and Pade approximant. It provides an analytical solution for the temperature distribution in the fin and explores the effects of variable thermal properties. Alhejaili et al. [21] presented an analytical solution for the temperature equation of a fin problem with variable temperature-dependent thermal properties. It applies the Least Squares Method (LSM) and DTM-Pade approximant to obtain the solution. The study investigates the thermal behavior of the fin with varying thermal properties. Khan et al. [22] analyzed heat transmission in a convective, radiative, and moving rod with thermal conductivity using a meta-heuristic-driven soft computing technique. It focuses on optimizing the heat transfer process using meta-heuristic algorithms and soft computing techniques. Sowmya et al. [23] examined the transient thermal distribution in a convective-radiative moving rod using the Two-Dimensional Differential Transform Method (2D DTM) with multivariate Pade approximant. It presents an analytical solution for the temperature distribution in the rod, considering both convective and radiative heat transfer. In summary, these manuscripts cover different aspects of thermal analysis and heat transfer problems. Jayaprakash et al. [20] focused on a trapezoidal fin, Alhejaili et al. [21] dealt with a fin problem, Khan et al. [22] explored heat transmission in a rod with meta-heuristic techniques, and Sowmya et al. [23] examined a convective-radiative moving rod. Each study applies different numerical methods and addresses specific variations and aspects of the thermal problems they investigate. In this analysis we provide some insights on what could potentially make this investigation on the thermal performance of a continuously moving convective-radiative rod with variable thermal conductivity novel, approach, scope, and results obtained. The analysis of the thermal performance of a continuously moving convective-radiative rod with variable thermal conductivity has significant application potential in various domains. Here are additional aspects of its applicability. The investigation of thermal performance in moving rods with temperature-dependent thermal conductivity is relevant in industrial processes such as continuous casting and rolling in the metal industry. Precise control of temperature distribution is crucial for ensuring product quality and optimizing production efficiency. In metallurgy and steel production, understanding how thermal conductivity varies with temperature in metal rods can enhance the quality and efficiency of steel products during continuous casting and rolling processes. Additionally, this research can be applied to the design and optimization of heat exchangers and thermal systems, leading to energy savings and improved performance. It can also find utility in materials processing, aiding in better temperature control within materials during manufacturing processes. In aerospace and automotive industries, where materials experience extreme temperature variations, insights into variable thermal conductivity can improve the design and performance of components like engine parts and heat shields. Moreover, the research can contribute to energy-efficient building materials and insulation systems, regulating indoor temperatures more effectively and reducing energy consumption for heating and cooling. Furthermore, the analysis of the thermal behavior of moving rods with variable thermal conductivity contributes to the optimization of heat transfer processes. By accurately modeling and understanding temperature distribution, engineers and researchers can identify areas for improvement, such as optimizing cooling systems, reducing energy consumption, and enhancing overall thermal efficiency. Additionally, the problem of thermal performance in moving rods with temperature-dependent thermal conductivity finds application in material processing techniques. It can assist in the design and improvement of heat treatment processes like annealing or quenching, where precise temperature control is necessary to achieve desired material properties. Moreover, the study of convective-radiative heat transfer in moving rods has implications for energy systems. Understanding thermal behavior and optimizing heat transfer in systems involving moving components can contribute to more efficient and sustainable energy conversion and transportation processes. The use of advanced analytical methods such as the Least Square method (LSM) and the Galerkin method (GM) to solve the nonlinear temperature distribution equation in moving rods also holds broader implications. These methods can be applied to various engineering problems involving partial differential equations and complex systems, providing accurate solutions and improving computational efficiency. In summary, the analysis of the thermal performance in continuously moving convective-radiative rods with variable thermal conductivity has wide-ranging applicability in industrial processes, heat transfer optimization, material processing, energy systems, and the advancement of advanced numerical methods. By addressing the challenges associated with temperature-dependent thermal conductivity in moving rods, this research offers valuable insights and solutions that can be applied across diverse fields, ultimately leading to enhanced performance, energy efficiency, and process optimization.

This investigation could be novel in terms of the scope of the problem addressed. The previous investigations focused on a particular aspect of the problem, such as the convective and radiation heat transfer, while the current study considers convective and radiative heat transfer on the longitudinal moving with variable thermal conductivity; this would be considered a novel contribution. This is because the metal is generally moving in the thermal processing of continuous casting and rolling. When doing a thermal performance analysis, the constant thermo-physical properties assumptions, such as thermal conductivity, cannot be appropriate if a significant temperature difference between the fin-tip and fin-base exists; variation of the thermal conductivity is very substantial and should be considered temperature-dependent. Besides, the cross-sections of the moving metal can be another topic of investigation rod, sheet, or other structural ones. Therefore, the abovementioned study considers the physical parameter for its specified conditions. This investigation addresses the thermal distribution through a continuously convective-radiative longitudinal moving fin with variable thermal conductivity. The investigation could be essential in terms of the results obtained. It reveals insights into the underlying physics of the problem, which significantly contributes to the field. The effects of six dimensionless parameters, including the dimensionless thermal conductivity coefficient, the convective-conductive parameter, the radiative-conductive parameter, the Peclet number, the dimensionless convective sink temperature, and the dimensionless radiative sink temperature on the temperature distribution are analyzed. Finally, the investigation is also important in terms of the approach used to study the problem with the mathematical complexity of the energy equation. The use of a powerful analytical and numerical method that allows for more accurate measurements could make the investigation stand out. In summary, the novelty of this investigation depends on various factors, including the approach, scope, and results obtained.

In this study, we have applied GM and LSM to find the approximate solutions of nonlinear differential equations governing the convective-radiative heat transfer of a continuously moving rod with temperature-dependent thermal conductivity. In addition, the effects of key parameters, including the dimensionless thermal conductivity coefficient $(a)$, the convective-conductive parameter $(N_{cc})$, the radiative-conductive parameter $(N_{rc})$, the Peclet number $(Pe)$, the dimensionless convective sink temperature $(\theta_c)$ and the dimensionless radiative sink temperature $(\theta_r)$, on the dimensionless temperature distribution are analyzed. Meanwhile, the results demonstrate that the proposed methods are simple and accurate compared with the numerical method. It is found that these methods are powerful mathematical tools and that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering

## 2. Analysis

As shown in Fig. 1, the thermal processing of a moving rod with a temperature-dependent thermal conductivity fin profile is considered. The dimensions of the fin are length $L$, width $W$ and thickness $t$. The cross-section area $A$ of the fin is constant. For the sake of simplify of the solution, the following assumptions are made to solve this problem. The velocity of the moving rod is $v$. The hot rod emerges from a hotter environment at a constant temperature $T_b$ to a colder temperature and releases heat in the surrounding medium, which is considered by convection and radiation.

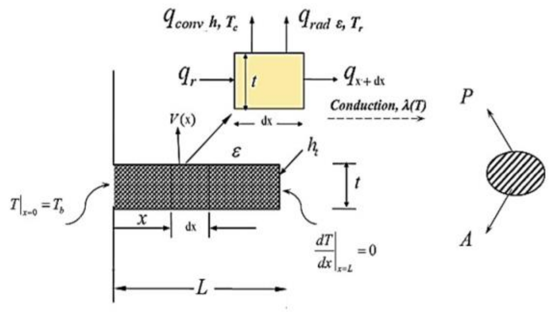

**Fig. 1.** Schematic diagram for the problem under consideration

The convection and radiation sink temperatures are taken to be different, and one can be varied independently of the other. The surface of moving material is assumed to be gray with a constant emissivity. Hence, we use the radiation formula for the gray body. The convective heat transfer coefficient h over the entire surface of the moving material is considered constant. Now, applying the energy balance equation at steady state energy equation of the moving rod with a constant speed and heat loss through natural convection and radiation condition [24] to the slice segment of the fin of thickness ΔX

$$\frac{d}{dx}\left[k\frac{dT}{dx}\right]-\frac{hp}{A}[T(x)-T_c]^2+\frac{\varepsilon\sigma p}{A}\left(T^4-{T_r}^4\right)+\rho c_p\vartheta\frac{dT}{dx}=0 \tag{1}$$

where $k$ is the thermal conductivity of the moving rod may vary with temperature, $\rho$ is the density, and $c_p$ is the specific heat. The axial coordinate $x$ is measured from the tip of the moving fin. There are Dirichlet and Neumann boundary conditions for the moving rod. Boundary conditions are

$$x=0 \quad \rightarrow \quad T(x)=0 \tag{2}$$

$$x=L \quad \rightarrow \quad \frac{dT(x)}{dx}=0 \tag{3}$$

Taking into account that the thermal conductivity of the fin k is assumed to vary with temperature:

$$k(T)=\lambda_c f(T)\,, \quad f(T)=1+\beta(T-T_c) \tag{4}$$

Introducing dimensionless variables and similarity criteria

$$X=\frac{xL^*}{L}\ ,\ L^*=\frac{PL}{A}\ ,\ \theta=\frac{T}{T_b}\ ,\quad \theta_c=\frac{T_c}{T_b}\ ,\theta_r=\frac{T_r}{T_b}\ ,\ N_{cc}=\frac{hA}{\lambda_c p}\ ,\quad N_{rc}=\frac{\sigma\varepsilon T_b^3 A}{\lambda_c p}\ ,\ Pe=\frac{\rho c_p v A}{\lambda_c p} \tag{5}$$

For the sake of simplicity, the dimensionless characteristic length is assumed as L*=1 in the following discussions. By substituting them into Eq. (9) and using Eq. (11), the problem formulation takes the following form:

$$[1+a(\theta-\theta_c)]\frac{d^2\theta}{dX}+a\left(\frac{d\theta}{dX}\right)^2-N_{cc}(\theta-\theta_c)-N_{rc}(\theta^4-\theta_r^4)+Pe\frac{d\theta}{dX}=0 \tag{6}$$

where $a$ denotes the dimensionless thermal conductivity coefficient, $N_{cc}$ is the convective-conductive parameter, $N_{rc}$ is the radiative-conductive parameter, $Pe$ the Peclet number, the $\theta_c$ and $\theta_r$ are the dimensionless convective sink temperature and the dimensionless radiative sink temperature, respectively. The above equation is a second-order nonlinear ordinary differential equation which is subject to the boundary conditions:

$$\theta(1)=1 \quad , \quad \theta'(0)=0 \tag{7}$$

## 3. Weighted Residual Methods (WRM)

There existed an approximation technique for solving differential equation called the weighted residual methods (WRMs). Suppose a differential operator $D$ is acted on a function $u$ to produce a function $p$[25, 26]:

$$D\big(u(x)\big)=p(x) \tag{8}$$

It is considered that $u$ is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \quad (9)$$

Now, when substituted into the differential operator, $D$, the result of the operations generally isn't $p(x)$. Hence an error or residual will exist:

$$E(x) = R(x) = D\big(\tilde{u}(x)\big) - p(x) \neq 0 \quad (10)$$

The notion in WRMs is to force the residual to zero in some average sense over the domain. That is:

$$\int_X R(x) W_i(x) = 0 \quad , \quad i = 1,2,\ldots,n \quad (11)$$

where the number of weight functions $W_i$ is exactly equal the number of unknown constants $C_i$ in $\tilde{u}$. The result is a set of $n$ algebraic equations for the unknown constants $C_i$. Two methods of WRMs are explained in the following subsections.

### *3.1. Shortcomings and Advantages of the Methodology*

The Galerkin Method is a powerful numerical technique that offers flexibility, accuracy, and conservation properties. It can be applied to various problems, including complex systems and partial differential equations (PDEs). By approximating the solution using a series of basis functions, the Galerkin Method can provide accurate results, with improved accuracy as the number of basis functions increases. Additionally, it has the ability to preserve certain properties of the original problem, such as mass conservation or energy conservation, depending on the chosen formulation. However, the Galerkin Method has its shortcomings. One major challenge is its computational complexity, especially when dealing with problems that have high-dimensional domains or complex geometries. Solving large systems of equations can lead to significant computational costs and limitations on the size of the problem that can be effectively solved. Furthermore, the convergence of the Galerkin Method depends on various factors, including the choice of basis functions, the regularity of the solution, and the problem's characteristics. In some cases, achieving convergence may be slow or even fail to reach the desired level of accuracy. Additionally, the Galerkin Method may encounter difficulties when dealing with nonlinear problems, as the approximation of the solution may not be accurate enough to capture nonlinear effects. In contrast, the Least Squares Method (LSM) is a versatile numerical method that is often preferred for its robustness and ease of implementation. It can handle a wide range of problems, including linear and nonlinear systems, as well as situations involving noisy or incomplete data. The simplicity of implementing the LSM makes it particularly suitable for problems with a small number of unknowns or a low-dimensional domain. Additionally, the LSM provides estimates of the error between the approximate solution and the true solution, allowing for an assessment of the solution's accuracy. However, the LSM also has its limitations. It is sensitive to outliers in the data because it aims to minimize the sum of squared errors. Outliers can significantly impact the solution, leading to inaccuracies. Another drawback is the lack of inherent conservation properties in the LSM. Unlike the Galerkin Method, which can preserve properties such as mass or energy conservation, the LSM requires additional techniques or constraints to enforce such properties. Furthermore, the LSM relies on assuming a functional form for the solution and seeks the best fit within that form. If the chosen form does not accurately represent the true solution, the LSM may introduce approximation bias and yield less accurate results. In summary, the Galerkin Method is suitable for problems requiring high accuracy and conservation properties, but it can be computationally demanding and may face convergence challenges [27, 28]. On the other hand, the LSM is robust, versatile, and easy to implement, but it may be sensitive to outliers and lacks inherent conservation properties [29, 30]. The choice between these methods depends on the specific problem, its complexity, and the desired trade-offs between accuracy, computational cost, and robustness.

The Least Square Method (LSM) and the Galerkin Method (GM) offer notable advantages over other numerical methods in the realm of mathematical and engineering problem-solving. LSM, by striving to minimize the error between approximate and actual solutions, consistently delivers high levels of accuracy. Its versatility enables it to handle a wide array of problems, irrespective of linearity or boundary conditions, while its grid-independence simplifies problem setup. Furthermore, LSM exhibits adaptability to complex geometries and robustness when dealing with ill-conditioned or noisy data. On the other hand, GM's universal applicability makes it a valuable choice for solving diverse differential equations, including partial differential equations (PDEs). Its systematic approach, employing basis functions that best represent the solution space, ensures accurate and stable results. GM also preserves essential physical quantities, like mass and energy, making it ideal for a wide range of engineering and scientific applications. Both methods are relatively straightforward to implement, offering researchers and engineers accessible tools for tackling complex problems effectively. LSM and GM methods possess distinct advantages over numerical techniques for problem-solving in mathematics and engineering. These methods offer exact solutions, ensuring the highest level of accuracy when applicable. They are known for their simplicity and transparency, providing clear insights into problem behaviors. Analytical solutions are computationally efficient, eliminating the need for time-consuming iterations and simulations. They also facilitate easy parameter sensitivity analysis, enabling a comprehensive understanding of how input variations impact system responses. Moreover, analytical solutions often yield general insights and mathematical relationships that can be broadly applied, reducing the need for recalibration. They do not introduce discretization errors, ensuring greater accuracy. However, it's important to acknowledge that analytical methods are best suited for well-structured problems and may not be feasible for complex, real-world scenarios where numerical methods are essential.

### *3.2. Application of Galerkin Method (GM)*

For GM, the derivative of the approximating function or trial function is used finding weighted function. In this method, weight functions are:

$$W_i = \frac{\partial \tilde{u}}{\partial c_i} \quad , \quad i = 1,2,\dots,n \tag{12}$$

In the present study, the governing equations of the convective-radiative transfer longitudinal moving rod are solved by GM, LSM and NUM. For solving the problem using GM, because the trial function must satisfy the boundary conditions in Eq. (7), so it will be considered as,

$$\theta(X) = 1 + c_1\left(X - \frac{1}{2}X^2\right) + c_2\left(X - \frac{1}{3}X^3\right) + c_3\left(X - \frac{1}{4}X^4\right) + c_4\left(X - \frac{1}{5}X^5\right) \tag{13}$$

By introducing them into Eq. (6), residual functions, R, will be found. On the other hand, the residual functions must be close to zero. To reach this aim, by using Eq. (7), weighted functions will be obtained as,

$$W_1 = X - \frac{1}{2}X^2 \;, W_2 = X - \frac{1}{3}X^3 \;, \quad W_3 = X - \frac{1}{4}X^4 \quad , \quad W_4 = X - \frac{1}{5}X^5 \tag{14}$$

Finally, by substituting these functions into Eq. (6), a set of four equations and four unknown coefficients will be obtained. After solving these unknown parameters, the temperature distribution will be determined. Using the Galerkin method when physical parameters are assumed to be $a = 0.5, N_{cc} = 0.25, N_{rc} = 0.75, Pe = 0.5$ , $\theta_c = 0.4$, and $\theta_r = 0.8$ leads to:

$$\theta(x) = 1 - 0.3118147X + 0.26749445X^2 - 0.123325X^3 + 0.046854X^4 - 0.008123X^5 \tag{15}$$

### *3.3. Principles and Application of Least Square Method (LSM)*

If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of [31, 32]:

$$S = \int_X R(x)R(x)d(x) = \int_X R^2(x)dx \tag{16}$$

In order to achieve a minimum of this scalar function, the derivatives of $S$ with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i} = 2\int_X R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{17}$$

Comparing with Eq. (17), the weight functions are seen to be

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{18}$$

however, the "2" coefficient can be dropped since it cancels out in the equation. Therefore, the weight functions for the least squares method are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{19}$$

Consider the following trial function,

$$\theta(X) = 1 + c_1\left(X - \frac{1}{2}X^2\right) + c_2\left(X - \frac{1}{3}X^3\right) + c_3\left(X - \frac{1}{4}X^4\right) + c_4\left(X - \frac{1}{5}X^5\right) \tag{20}$$

The residual function is:

$$\begin{aligned} R(X) = \frac{1}{8}[&-4ac_2^2 - 4ac_2c_3 - 12N_{rc}c_1c_3 \\ &-6N_{rc}c_2^2 - 12N_{rc}c_2c_3 - 6N_{rc}c_3^2 - 3c_3 \\ &-12N_{rc}c_1c_2 - 4ac_1c_2 - 6N_{rc}c_1^2 - Pec_2 \\ &-12N_{rc}c_2c_4 - 6N_{rc}c_4^2 + \frac{3}{2}ac_1^2 - 12N_{rc}c_1c_4 \\ &+\frac{1}{2}N_{cc}c_1 + 2N_{rc}c_1 - 12N_{rc}c_3c_4 \\ &+3a\theta_c c_3 - 3ac_3 - 4ac_2c_4] \end{aligned} \tag{21}$$

$$\times\left[\frac{12}{25}N_{rc}c_1c_2c_4+\frac{18}{25}N_{rc}c_1c_4^2+\frac{12}{25}N_{rc}c_1^2c_4\right.$$
$$+\frac{4}{5}N_{rc}c_2c_3c_4+\frac{2}{5}N_{rc}c_1c_2c_3+\frac{1}{16}N_{rc}c_3^3$$
$$+\frac{2}{5}N_{rc}c_2^2c_3+\frac{3}{20}N_{rc}c_3^2+\frac{12}{25}N_{rc}c_1c_3c_4$$
$$\left.-\frac{3}{20}N_{rc}c_1^2c_3-\frac{2}{15}N_{rc}c_1c_2^2+\frac{2}{5}N_{rc}c_2c_3^2\right]$$
$$\times 4ac_4X^3+\cdots+\frac{9}{5}N_{rc}c_1X^8c_2c_3c_4$$
$$+2N_{rc}c_1X^7c_2c_3c_4-8N_{rc}c_1X^6c_2c_3c_4$$
$$-\frac{3}{5}N_{rc}c_1X^{10}c_2c_3c_4-\frac{17}{6}N_{rc}c_1X^8c_2^2c_3=0$$

After introducing $a=0.5$, $N_{cc}=0.25$, $N_{rc}=0.75$, $Pe=0.5$, $\theta_c=0.4$, and $\theta_r=0.8$ coefficients to residual function, Eq. (21), C1 till C4 coefficients will be calculated and temperature distribution will be:

$$\theta(X)=1-0.3119294X+0.26832437X^2$$
$$-0.125214X^3 0.048583X^4-0.0086821X^5 \tag{22}$$

## 4. Results and Discussion

In the present study, analytical techniques called the least Square method (LSM) and Galerkin method (GM) are applied to obtain an explicit solution of the moving rod fin temperature-dependent heat conduction. First, a comparison between the applied methods, obtained by the GM, LSM, and numerical method for different values of active parameters is shown in Figures 2 till 6. The numerical solution is performed using the algebra package Maple 18.0 to solve the present case. The package uses a boundary value (B-V) problem procedure [33].

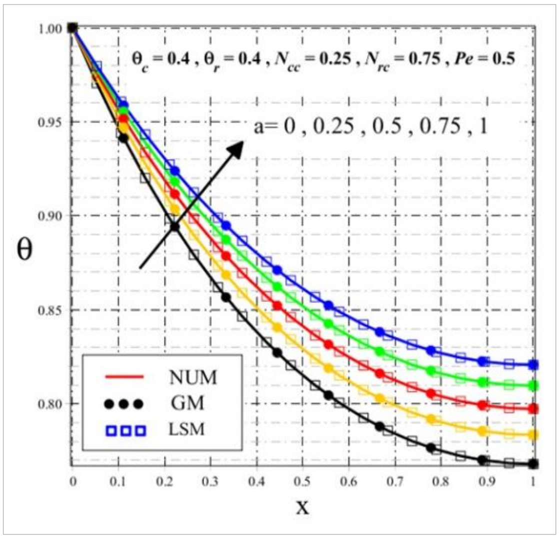


**Fig. 2.** Effect dimensionless thermal conductivity coefficients ($a$) and comparison of GM, LSM results with the numerical solution

The algorithm can be used to find moderate accuracy solutions for ODE boundary value problems and initial value problems, both with a global error bound. The method uses either Richardson extrapolation or deferred corrections with a base method of either the trapezoid or midpoint method. The trapezoid method is generally efficient for typical problems, but the midpoint method is so capable of handling harmless end-point singularities that the trapezoid method cannot. The midpoint method, also known as the fourth-order Runge–Kutta–Fehlberg method, improves the Euler method by adding a midpoint in the step, increasing the accuracy by one order. Thus, the midpoint method is used as a suitable numerical technique [34]. The results are proven to be precise and accurate in solving a wide range of mathematical and engineering problems, especially heat transfer cases [35]. Comparing the analytical solution with the numerical results highlights that the suggested approaches are effective and robust tools for addressing heat transfer challenges. Furthermore, this comparison demonstrates that there is minimal disparity in the temperature profiles when comparing the two methods, namely LSM and GM. This research has been thoroughly examined, showcasing the impact of parameters such as the convective-conductive factor ($N_{cc}$), dimensionless thermal conductivity coefficients ($a$), radiative-conductive parameter ($N_{rc}$), Peclet number ($Pe$), and dimensionless convective ($\theta_c$) and radiative sink temperatures ($\theta_r$) on heat transfer.

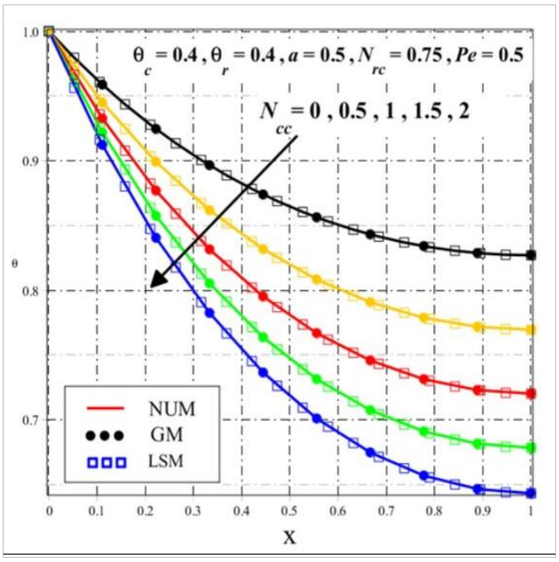


**Fig. 3.** Effect of convective-conductive parameter($N_{cc}$) and comparison of GM, LSM results with the numerical solution

Figure 2 displays the temperature distribution in the moving rod under different thermal conductivity coefficients. As depicted in Figure 2, an increase in the dimensionless thermal conductivity coefficient ($a$) corresponds to a rise in the distribution of dimensionless temperature within the moving rod. This escalation in thermal conductivity coefficient amplifies the conductive heat transfer, resulting in a subsequent increase in temperature distribution. Furthermore, Figures 3 and 4 illustrate the fluctuations in dimensionless temperature associated with varying values of the convective-conductive and radiative-conductive parameters. The convective-conductive parameter (Ncc) represents the ratio of convective heat transfer to conductive heat transfer within the moving rod.

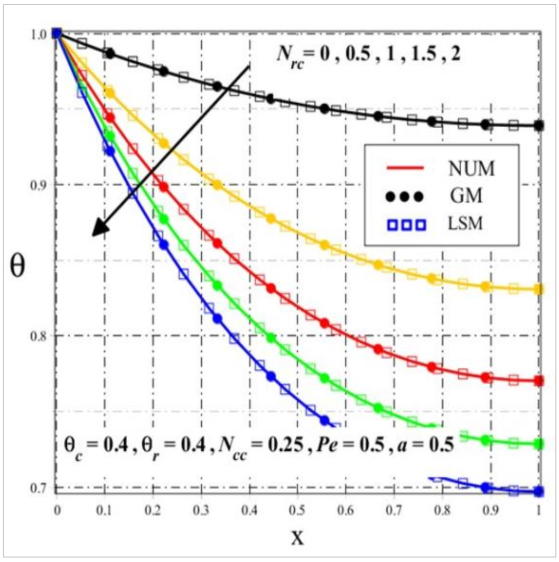


**Fig. 4.** Effect of radiative-conductive parameter ($N_{rc}$)and comparison of GM, LSM results with the numerical solution

Consequently, an elevation in the convective-conductive parameter corresponds to an increased magnitude of heat loss. Likewise, the impact of the radiative-conductive parameter (Nrc) on dimensionless temperature is depicted in Figure 4. This visual representation illustrates that with an increase in radiative heat transfer, indicated by the rise in Nrc, the local temperature within the fin experiences a decrease. Furthermore, Figure 5 illustrates the impact of the Peclet number (Pe) on the distributions of dimensionless temperature along the moving rod. Notably, it is observed that the dimensionless temperature tends to increase as the Peclet number rises. In the context of the moving rod, the Peclet number is defined as the ratio of the thermal advective transport rate to the thermal diffusive transport rate. An increase in the Peclet number implies that the rod is moving at a higher speed, consequently resulting in higher dimensionless temperatures. Figures 6 and 7 provide a detailed depiction of how changes in dimensionless convective sink temperatures and dimensionless radiative sink temperatures influence temperature profiles. An increase in convective sink temperatures and dimensionless radiative sink temperatures results in higher dimensionless temperatures within a fin. Analyzing Figure 6 reveals a clear link between raising the convective sink temperature and a significant drop in convective heat loss from the moving rod. This observation emphasizes the direct connection between the convective sink temperature and the heat transfer mechanisms occurring within the rod.

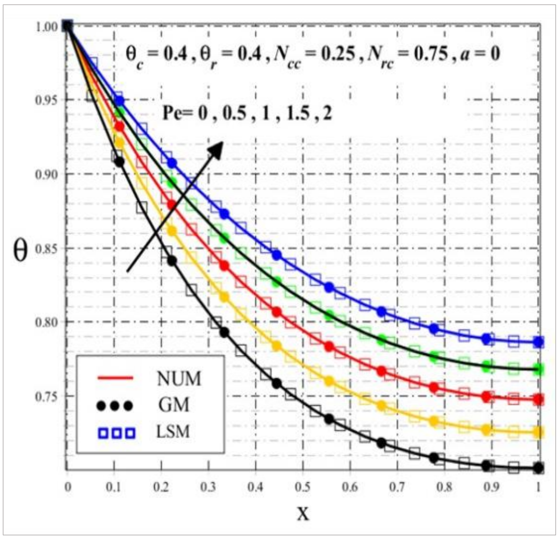


**Fig. 5.** Effect of Peclet number (Pe) and comparison of GM, LSM results with the numerical solution

This decrease in convective heat loss can lead to an uptick in the dimensionless temperature within the moving rod. Conversely, when turning our focus to Figure 7, increasing the radiative sink temperature initiates a contrasting effect-a decrease in radiative heat loss and, consequently, an increase in the dimensionless temperature within the moving rod.

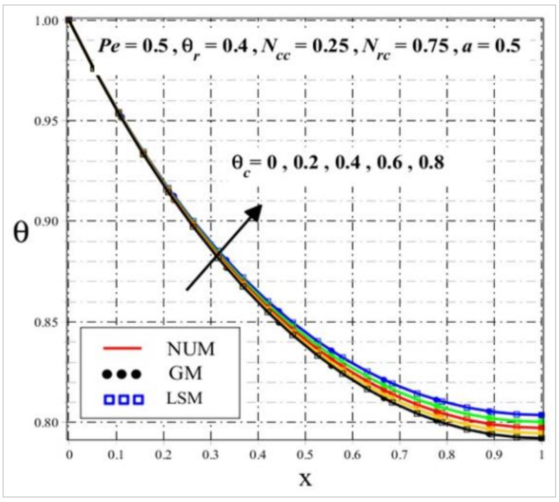


**Fig. 6.** Effect of dimensionless convective sink temperatures *($\theta_c$)* and comparison of GM, LSM results with the numerical solution

It's important to emphasize that when we compare the impacts of these two sink temperatures, the dimensionless radiative sink temperature emerges as the more influential factor in determining the overall dimensionless temperature within the moving rod. This underscores the intricate interplay of factors governing heat transfer and temperature distribution in this system.

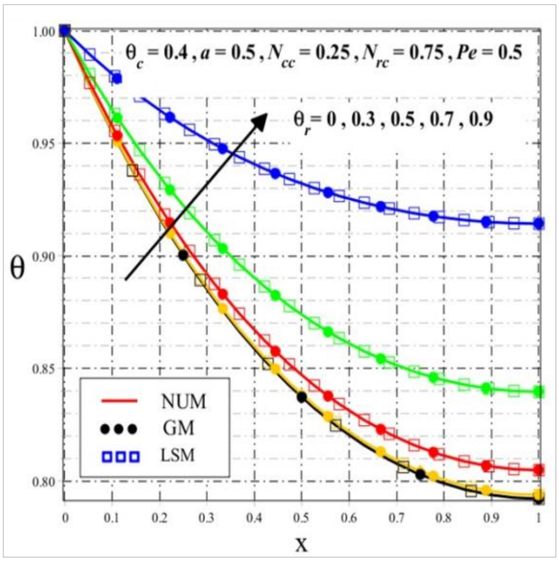


**Fig. 7.** Effect dimensionless radiative sink temperatures ($\theta_r$)and comparison of GM, LSM results with the numerical solution

## 5. Conclusions

This study investigates rod fins' thermal performance with temperature-dependent thermal conductivity in detail via the Galerkin Method (GM) and Least Square Method (LSM). Dimensionless temperature distribution along the length of the fin has been determined as a function of convective, radiative, and Peclet numbers. Also, the fourth-order runge-kutta method is applied as a numeric scheme to tackle and show the high ability of proposed methods on nonlinear systems of the ordinary differential equations.

The following important points can be concluded from the present study:

The comparison of the analytical solution with the numerical outcomes shows that the proposed methods are convenient and powerful methods in heat transfer problems. Also, the comparison revealed that the difference in the temperature profiles is almost negligible when the two methods (LSM and GM) are compared. Moreover, increasing the thermal conductivity coefficient will enhance the conductive heat transfer, and this will lead to an increase in the distribution of temperature. In addition, the results indicate that as the buoyancy effects become stronger, i.e., Ncc increases, the local temperature in the fin decreases. Similarly, the local fin temperature decreases as the increases of radiative heat exchange between the exposed surface of the fin and the ambient. Moreover, the results illustrate that the dimensionless radiative sink temperature has a more significant role in the dimensionless temperature than the dimensionless convective sink temperature. A rise in the Peclet number, denoting the ratio of thermal advective transport rate to thermal diffusive transport rate in the context of the moving rod, correlates with elevated dimensionless temperatures, signifying that greater rod velocities result in higher temperatures. Looking ahead, there are several potential avenues for future research. One promising direction is to delve deeper into the analysis of entropy generation within convective-radiative moving fins, taking into account variables like variable thermal conductivity and internal heat source. Furthermore, our research will explore the thermal performance of fins with various profiles under the influence of magnetic field effects. These investigations will expand our understanding of heat transfer in complex scenarios and could have practical implications for the design and optimization of thermal systems


## Funding Statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.


## Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

## References


[1] Kiwan, S. and Al-Nimr, M. A., 2001. Using porous fins for heat transfer enhancement. *Journal of Heat Transfer*, 123(4), pp.790-795.

[2] Kiwan, S. and Zeitoun, O., 2008. Natural convection in a horizontal cylindrical annulus using porous fins. *International Journal of Numerical Methods for Heat & Fluid Flow*. 18(5), pp.618-634.

[3] Nayfeh, A. H., 2008. *Perturbation methods*. John Wiley & Sons.

[4] Ganji, D. D., Kachapi, S. H. and Seyed, H., 2011. Analytical and numerical methods in

engineering and applied sciences. *Progress in Nonlinear Science*, 3, pp.1-579.

[5] Ganji, D. D. and Kachapi, S. H., 2011. Analysis of nonlinear equations in fluids. *Progress in Nonlinear Science*, 2, pp.1-293.

[6] He, J. H., 2005. Homotopy perturbation method for bifurcation of nonlinear problems. *International Journal of Nonlinear Sciences and Numerical Simulation*, 6(2), pp. 207-208.

[7] Ganji, D. D., Abbasi, M., Rahimi, J., Gholami, M., Rahimipetroudi, I., 2014. On the MHD squeeze flow between two parallel disks with suction or injection via HAM and HPM. *Frontiers of Mechanical Engineering*, 9, pp. 270-280.

[8] Abbasi, M., Ganji, D. D., Rahimipetroudi, I., Khaki, M., 2014. Comparative analysis of MHD boundary-layer flow of viscoelastic fluid in permeable channel with slip boundaries by using HAM, VIM, HPM. *Walailak Journal of Science and Technology (WJST)*, 11(7), pp. 551-567.

[9] He, J. H., 2007. Variational iteration method—some recent results and new interpretations. *Journal of computational and applied mathematics*, 207(1), pp. 3-17.

[10] Petroudi, R. I., Ganji, D. D., Shotorban, B. A., Nejad, K. M., Rahimi, E., Rohollahtabar, R., Taherinia, F., 2012. Semi-analytical method for solving non-linear equation arising of natural convection porous fin. *Thermal Science*, 16(5), pp. 1303-1308.

[11] He, J. H., 1999. Variational iteration method–a kind of non-linear analytical technique: some examples. *International journal of non-linear mechanics*, 34(4), pp.699-708.

[12] Vahabzadeh, A., Fakour, M., Ganji, D. D., Rahimipetroudi, I., 2014. Analytical accuracy of the one dimensional heat transfer in geometry with logarithmic various surfaces. *Central European Journal of Engineering*, 4, pp.341-351.

[13] Liao, S., 2004. On the homotopy analysis method for nonlinear problems. *Applied mathematics and computation*, 147(2), pp. 499-513.

[14] Petroudi, I. R., Ganji, D. D., Nejad, M. K., Rahimi, J., Rahimi, E., Rahimifar, A., 2014. Transverse magnetic field on Jeffery–Hamel problem with Cu–water nanofluid between two non parallel plane walls by using collocation method. *Case Studies in Thermal Engineering*, 4, pp.193-201.

[15] Hasankhani Gavabari, R., Abbasi, M., Ganji, D. D., Rahimipetroudi, I., Bozorgi, A., 2016. Application of Galerkin and Collocation method to the electrohydrodynamic flow analysis in a circular cylindrical conduit. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 38, pp. 2327-2332.

[16] Kumar, R. V., Sarris, I. E., Sowmya, G., Abdulrahman, A., 2023. Iterative Solutions for the Nonlinear Heat Transfer Equation of a Convective-Radiative Annular Fin with Power Law Temperature-Dependent Thermal Properties. *Symmetry*, 15(6), pp. 1204.

[17] Hoshyar, H.A., Rahimipetroudi, I., Ganji, D.D., Majidian, A.R., 2015. Thermal performance of porous fins with temperature-dependent heat generation via the homotopy perturbation method and collocation method. *Journal of Applied Mathematics and Computational Mechanics*, *14*(4), pp.53-65.

[18] Varun Kumar, R. S., Alsulami, M. D., Sarris, I. E., Prasannakumara, B. C., Rana, S., 2023. Backpropagated neural network modeling for the non-fourier thermal analysis of a moving plate. *Mathematics*, 11(2), pp. 438.

[19] Sowmya, G., Kumar, R. S. V. and Banu, Y., 2023. Thermal performance of a longitudinal fin under the influence of magnetic field using Sumudu transform method with pade approximant (STM-PA). *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, pp. e202100526.

[20] Jayaprakash, M. C., Alzahrani, H. A., Sowmya, G., Kumar, R. V., Malik, M. Y., Alsaiari, A., Prasannakumara, B. C., 2021. Thermal distribution through a moving longitudinal trapezoidal fin with variable temperature-dependent thermal properties using DTM-Pade approximant. *Case Studies in Thermal Engineering*, 28, pp. 101697.

[21] Alhejaili, W., Kumar, R.V., El-Zahar, E.R., Sowmya, G., Prasannakumara, B.C., Khan, M.I., Yogeesha, K.M., Qayyum, S., 2022. Analytical solution for temperature equation of a fin problem with variable temperature-dependent thermal properties: Application of LSM and DTM-Pade approximant. *Chemical Physics Letters*, *793*, pp.139409.

[22] Khan, N. A., Sulaiman, M. and Alshammari, F. S., 2022. Analysis of heat transmission in convective, radiative and moving rod with thermal conductivity using meta-heuristic-driven soft computing technique. *Structural and Multidisciplinary Optimization*, 65 (11), pp. 317.

[23] Sowmya, G., Sarris, I. E., Vishalakshi, C. S., Kumar, R. S. V., Prasannakumara, B. C., 2021. Analysis of transient thermal distribution in a convective–radiative moving rod using two-dimensional differential transform method with multivariate pade approximant. *Symmetry*, 13(10), pp. 1793.

[24] Sun, Y. S., Ma, J. and Li, B. W., 2015. Spectral collocation method for convective–radiative transfer of a moving rod with variable thermal conductivity. *International Journal of Thermal Sciences*, 90, pp. 187-196.

[25] Hatami, M., Hasanpour, A. and Ganji, D. D., 2013. Heat transfer study through porous fins (Si3N4 and AL) with temperature-dependent heat generation. *Energy Conversion and Management*, 74, pp. 9-16.

[26] Ghasemi, S. E., Hatami, M., and Ganji, D. D., 2014. Thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation. *Case Studies in Thermal Engineering*, 4, pp. 1-8.

[27] Ghiaasiaan, S., 2010. *Applied Gas Dynamics, Chapter 3 provides an overview of the Galerkin Method and its advantages and limitations in solving fluid dynamics problems*. Cambridge University Press.

[28] Atkinson, K., 2018. *An Introduction to Numerical Analysis, Chapter 8 discusses the Galerkin Method, highlighting its strengths, computational complexity, and convergence issues*. CRC Press.

[29] Mukherjee, S., 2021. *Numerical Methods in Engineering with Python, Chapter 9 provides an introduction to the Galerkin Method and the LSM, presenting their advantages and shortcomings, along with practical examples*. CRC Press.

[30] Quarteroni, A., Saleri, F. and Gervasio, P., 2019. *Scientific Computing with MATLAB and Octave, Chapter 7 discusses the Galerkin Method and the LSM, addressing their advantages, computational complexity, and convergence issues, with MATLAB and Octave code examples*. Berlin: Springer.

[31] Hatami, M. and Ganji, D.D., 2013. Thermal performance of circular convective–radiative porous fins with different section shapes and materials. *Energy Conversion and Management*, 76, pp. 185-193.

[32] Hatami, M., Ahangar, G. R. M., Ganji, D. D., Boubaker, K., 2014. Refrigeration efficiency analysis for fully wet semi-spherical porous fins. *Energy conversion and management*, 84, pp. 533-540.

[33] Shirkhani, M. R., Hoshyar, H. A., Rahimipetroudi, I., Akhavan, H., Ganji, D. D., 2018. Unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). *Propulsion and Power Research*, 7(3), pp. 247-256.

[34] Hoshyar, H. A., Rahimipetroudi, I. and Ganji, D. D., 2019. Heat Transfer Performance on Longitudinal Porous Fins with Temperature-Dependent Heat Generation, Heat Transfer Coefficient and Surface Emissivity. *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, 43(2), pp. 383-391.

[35] Aziz, A., 2006. *Heat conduction with maple*. Philadelphia (PA): RT Edwards.