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## Research Article

# Bio-convective Magnetohydrodynamic Flow of Tangent Hyperbolic Nanofluid over a Stretching Surface with Convective Heat and Slip Conditions

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## ABSTRACT

The current study investigates the mixed convective flow of MHD tangent hyperbolic nanofluid due to a stretching surface with motile micro-organisms via convective heat transfer and slip conditions. The flow analysis's governing equations were converted into a non-dimensional relation by using the proper alteration. The PDE model equations are computed for these transformed equations using the MATLAB- bvp4c scheme. Skin friction, Sherwood number, Nusselt number, and the profiles of motile microorganisms are engineering-relevant quantities when compared to various physical variables. In comparison to recent literature, Skin friction is consistent for magnetic parameter; the results demonstrated a good consistency. Furthermore, an enhancement in the radiation and mixed convection parameter's magnitude enhances the velocity profile. Weissenberg number and magnetic field are used to study the reverse impact. The impact of thermal radiation parameter, Brownian movement, and thermophoretic effects are additional factors that frequently improve heat transfer. Through graphical and tabular explanations, the physical interpretation has been presented.

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## 1. Introduction

A fascination was brought on by the recent research on non-Newtonian fluids. Several researchers work in the mechanical, chemical, and technical fields in varying capacities. The easiest ways to come into contact with non-Newtonian fluids include polymeric relationships, atomic reactors, solvents, biomedical tumors, and many more. Non-Newtonian fluids might result in a non-linear correlation between shear rate and shear stress due to the complexity of their design and subsequent control using different types of models. Contrary to the Newtonian fluids, these

fluids are so complicated that it is impossible to comprehend their rheological characteristics by a simple connection. Several researchers have studied the flow of non-Newtonian fluids, such as Ali et al. [1], Alhadhrami et al. [2], Prameela et al. [3], Kumar et al. [4], Eswaramoorthi et al. [5], Mirzaee et al. [6], Das et al. [7], etc. by considering different fluid model. The tangent hyperbolic fluid is one of the most well-known examples of a non-Newtonian fluid. The results show that such a structure predicts the thinning characteristic arising from shear with high accuracy. To be more precise, any substance that exhibits a pattern of shear-thinning characteristics may be examined using the tangent hyperbolic fluid

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model. This fluid model is also known as the four-constant fluid model. In laboratories and industries, tangent hyperbolic fluids are often used. Some research items on tangent hyperbolic fluids are Ibrahim [8], Khan et al. [9], Bibi et al. [10], Salmi et al. [11], Faizan et al. [12], etc.

The fluid flow requires careful consideration of viscous dissipation when the fluid is moving at high speeds, raising its temperature. Typically, viscous dissipation is described as the variation of kinetic energy to internal energy, causing the fluid to become warmer because of viscosity, which consists of kinetic energy generated by turbulent motion and average flow kinetic energy. Nowadays, lots of research has been done on viscous dissipation, such as Gayatri et al. [13], Gopal et al. [14], Nalivela et al. [15], Li et al. [16], Das and Majumdar [17], Das et al. [18], etc.

The passage of an electric current through fluids, such as liquids and gases, can also cause joule heating. In this context, the term "joule heating" describes the procedure by which heat energy is produced by converting electrical energy inside the fluid medium. Joule heating in fluids is based on principles similar to those seen in solid conductors, with a few more factors to take into account. In order to move charges through a fluid, an electric current must overcome resistance. This resistance is largely affected by the fluid's electrical conductivity, which is controlled by elements, including its composition and temperature. Heat is produced as a result of energy transfer caused by collisions between the moving electrons and the fluid's molecules. Ahmed et al. [19] did research showing the variations of joule heating on a Maxwell nanofluid flow passing through a slender surface. Some recent researches on joule heating are Waqas et al. [20], Naseem et al. [21], Chamkha et al. [22], etc.

The term "bioconvection" describes convective motions that take the form of patterns when swimming microorganism density moves randomly in the form of cell colonies or certain cell types. Bio-convection is caused by microorganism density, which migrates upward and are denser than water. Denser top walls created unbalanced suspensions when microorganisms sank to promote bioconvection. Several studies on bioconvection in nanofluids have been published recently. Researchers in this subject are interested in a number of applications of bioconvection employing microbes and nanoparticles in the fields of biosensors, biotechnology, mechanical energy, manufacturing, and bioinformatics. Microbes swim in the top zone because the structure is irregular and unstable. Because of the high density of stratification, the top area becomes imbalanced when the current

microorganism swims on the higher surface. In marine habitats, such as river ponds, canyons, and seas, microbes lack constraints. Bioconvection has shown that microorganisms are swimming bacteria. The goal of bioconvection nano-liquids is to concentrate research on the impulsive configuration evolution and density stratification produced by simultaneously utilizing denser, self-driven microorganisms, nanoparticles, and drag forces. Working microorganisms may have organisms that exhibit gyrotaxis, gravitaxis, or oxytaxis. The inclusion of gyrotactic microorganisms in nano liquids enhances mass transference, microscale mixing, and nanofluid stability totally within micro volumes. Sajid et al. [23] show the influence of magnetohydrodynamics bioconvection tangent hyperbolic nanofluid on double-diffusive convection along motile microorganisms. Some other recent research on bio-convection are Shi et al. [24], Waqas et al. ([25],[26]), Shah et al. [27], Hussain et al. [28], Rashid et al. [29], Kaswan et al. [30], Manai et al. [31], etc.

The aforementioned survey shows that no attempt has been made to look at bioconvection analysis on the magnetized flow of tangent hyperbolic nanofluid taking viscous dissipation along joule heating into consideration. Since viscous dissipation and joule heating are involved, our main goal is to carefully examine the numerical treatment of the bio-convection flow of tangent hyperbolic nanofluid. Additionally, the effects of velocity slips are offset. The MATLAB bvp4c tool is used to produce the numerical results. It is covered in detail how different parameters' graphical trends compare to subjective flow fields.

## 2. Mathematical Formulation

To represent the flow of a non-Newtonian fluid, we must first understand its constitutive behaviour. In most non-Newtonian fluids, the constitutive relationship between the deviatoric stress tensor  $T$  and the applied strain rates  $E$  may be characterized by a time-independent scalar function  $\mu = \mu(\dot{\gamma})$  such that  $T = 2\mu(\dot{\gamma})E$ .

Here  $\mu$  is the generalized velocity which depends on first principal invariant  $\dot{\gamma} = \frac{1}{2}\sqrt{E : E}$  of the stress-strain rate tensor  $E$ .  $\dot{\gamma}$  reduces to the shear rate in the basic shear flow scenario. There have been several functional forms proposed for  $\mu(\dot{\gamma})$ , the most popular ones being the power-law model, the Carreau model, the Cross model, and the Herschel-Bulkley model.

In two-dimensional tangent hyperbolic nanofluid MHD, the flow of a motile microorganism over a stretching surface is conferred about. In this study, activation energy, nonlinear thermal radiation and convective boundary conditions were used, which can be used to enhance the rate of heat transfer. With a stream velocity of  $u_w$ , the flow would be perpendicular to the plane  $y \geq 0$  (figure 1). The microorganism swims independently due to the direction of nanoparticles. This study uses the Cartesian coordinate in its mathematical formulation. The applied magnetic field is in the transverse direction of the plate.

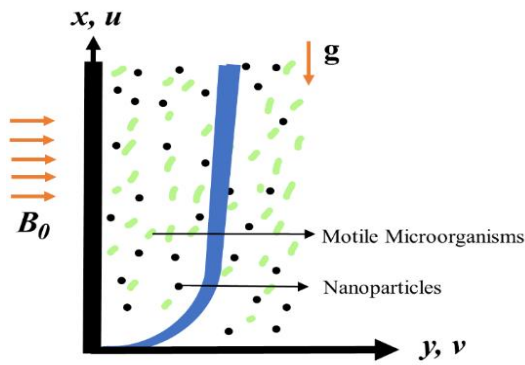


Fig. 1. physical model

The constitutive equations for the model's flow are as follows (Ibrahim [8], Khan et al. [9]).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \begin{aligned} &v(1-n) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \\ &+ \sqrt{2\nu n} \Gamma \left( \frac{\partial u}{\partial y} \right) \left( \frac{\partial^2 u}{\partial y^2} \right) \\ &+ g\beta(1-C_\infty)(T-T_\infty) \\ &- \frac{\gamma_1 g(\rho_m - \rho_f)(N - N_\infty)}{\rho_f} \\ &- \frac{g(\rho_p - \rho_f)(C - C_\infty)}{\rho_f} \end{aligned} \right) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \tau \left[ D_B \left( \frac{\partial C}{\partial y} \right) \left( \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] = \left( \begin{aligned} &\left( \alpha_f \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k_f(\rho C_p)_f} \right) \frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 \\ &+ \frac{\sigma B_0^2}{\rho C_p} u^2 \end{aligned} \right) \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - K_0(C - C_\infty) + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_M \frac{\partial^2 N}{\partial y^2} - b \frac{W_c}{(C_w - C_\infty)} \frac{\partial}{\partial y} \left( N \frac{\partial C}{\partial y} \right) \tag{5}$$

The boundary restrictions are

$$\left. \begin{aligned} &u = u_w + U_{slip}, v = 0, D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0, \\ &-k \frac{\partial T}{\partial y} = h_f(T_f - T), N = N_w \quad \text{at } y = 0 \\ &u \rightarrow 0, v \rightarrow 0, C \rightarrow C_\infty, N \rightarrow N_\infty, \\ &T \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \end{aligned} \right\} \tag{6}$$

The slip velocity  $U_{slip}$  is considered as (Wu [32])

$$U_{slip} = H \frac{\partial u}{\partial y} + S \frac{\partial^2 u}{\partial y^2} \tag{7}$$

here  $H$  and  $S$  are constant.

Consider the transformations

$$\left. \begin{aligned} &\eta = y \sqrt{\frac{c}{\nu}}, u = cx f'(\eta), \psi = \sqrt{c\nu} f(\eta), \\ &v = -\sqrt{c\nu} f(\eta), \theta(\eta) = \frac{T - T_w}{T_w - T_\infty}, \\ &\phi(\eta) = \frac{C - C_w}{C_w - C_\infty}, \chi = \frac{N - N_w}{N_w - N_\infty} \end{aligned} \right\} \tag{8}$$

Using the transformations (8), the eqs. (2)-(5) becomes

$$(1-n)f''' + nWi f f'''' - f'^2 + ff'' - Mf' + \lambda(\theta - Nr\phi - Rb\chi) = 0 \tag{9}$$

$$\left( 1 + \frac{4}{3} Rd \right) \theta'' + Pr \left( f\theta' + Nb\theta'\phi' + Nt\theta'^2 \right) + Ec f \phi'^2 + Ec M f'^2 = 0 \tag{10}$$

$$\phi'' + Pr Lef\phi' + \frac{Nt}{Nb} \theta'' - Pr Le\sigma^* \phi = 0 \tag{11}$$

$$\chi'' + Lbf\chi' - Pe(\chi'\phi' + \chi\phi'' + \delta_1\phi'') = 0 \tag{12}$$

The boundary restrictions are

$$\left. \begin{aligned} &f(\eta) = 0, f'(\eta) = 1 + \alpha f''(\eta) + \beta f'''(\eta), \\ &\theta'(0) = Bi(\theta(0) - 1), Nb\theta'(\eta) + Nt\phi'(\eta) = 0, \\ &\chi(\eta) = 1 \quad \text{at } \eta = 0 \\ &f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0, \chi \rightarrow 0 \quad \text{at } \eta \rightarrow \infty \end{aligned} \right\} \tag{13}$$

Where

$$\left. \begin{aligned} Wi &= \frac{\sqrt{2c^3} x \Gamma}{\sqrt{\nu}}, \lambda = \frac{g \beta (T_w - T_\infty)(1 - C_\infty)}{c^2 x}, \\ Rb &= \frac{\gamma_1 (N_w - N_\infty)(\rho_m - \rho_f)}{\rho_f \beta (T_w - T_\infty)(1 - C_\infty)}, Lb = \frac{\nu}{D_M}, \\ Nr &= \frac{(\rho_p - \rho_f)(C_w - C_\infty)}{\rho_f \beta (T_w - T_\infty)(1 - C_\infty)}, Pe = \frac{bW_c}{D_M}, \\ Nb &= \frac{\tau D_B (C_w - C_\infty)}{\nu}, Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}, \\ M &= \frac{\sigma B_0^2}{\rho c}, Rd = \frac{4T_\infty^3}{k_f (\rho C_p)_f}, Pr = \frac{\nu}{\alpha_f}, \\ Le &= \frac{\alpha_f}{D_B}, Ec = \frac{c^2 x^2}{(T_w - T_\infty) C_p}, \sigma^* = \frac{K_0}{c} \\ \delta_1 &= \frac{N_\infty}{N_w - N_\infty}, \end{aligned} \right\} \quad (14)$$

Skin friction, Sherwood number, Nusselt number, and microorganism Sherwood number are as follows

$$\left. \begin{aligned} C_f &= \frac{2\tau_w}{\rho u_w^2}, Sh_x = \frac{xq_m}{D_M (C_w - C_\infty)}, \\ Nu_x &= \frac{xq_w}{\kappa (T_w - T_\infty)}, Nn_x = \frac{xq_n}{D_m (N_w - N_\infty)} \end{aligned} \right\} \quad (15)$$

where  $\tau_w$  is denoted for shear stress and heat flux is

$$q_w = -\kappa \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (16)$$

In the nondimensional form the equation (15) are as follows

$$\left. \begin{aligned} C_f \sqrt{Re_x} &= \left( (1-n)f''(0) + \frac{n}{2} Wif''(0)^2 \right), \\ \frac{Sh_x}{\sqrt{Re_x}} &= -\phi'(0), \\ \frac{Nu_x}{\sqrt{Re_x}} &= -\left( 1 + \frac{4}{3} Rd \right) \theta'(0), \\ \frac{Nn_x}{\sqrt{Re_x}} &= -\chi'(0), \end{aligned} \right\} \quad (17)$$

### 3. Solution Methodology

The set of the highly nonlinear, non-dimensional equation (9-12) subjected to boundary restriction (13) is a two-point boundary-value problem. A well-known tool

called `bvp4c` is used under MATLAB to solve the equations.

Therefore, to implement `bvp4c`, paired non-dimensional differential equations with boundary restrictions must be reduced into a first-order simultaneous equations system. Let

$$\left. \begin{aligned} f &= \Gamma_1, f' = \Gamma_2, f'' = \Gamma_3, f''' = \Gamma_3', \\ \theta &= \Gamma_4, \theta' = \Gamma_5, \theta'' = \Gamma_5', \\ \phi &= \Gamma_6, \phi' = \Gamma_7, \phi'' = \Gamma_7', \\ \chi &= \Gamma_8, \chi' = \Gamma_9, \chi'' = \Gamma_9', \\ \Gamma_3' &= \frac{\left( \Gamma_2^2 + M \Gamma_2 - \Gamma_1 \Gamma_3 \right)}{1 - n + nWi \Gamma_3}, \\ \Gamma_5' &= \frac{-Pr \left( \Gamma_1 \Gamma_5 + Nb \Gamma_5 \Gamma_7 + Nt \Gamma_5^2 \right) + Ec \Gamma_3^2 + Ecm \Gamma_2^2}{\left( 1 + \frac{4}{3} Rd \right)}, \\ \Gamma_7' &= -\frac{Nt}{Nb} \Gamma_5' - Pr Le \Gamma_1 \Gamma_7 + Pr Le \sigma^* \Gamma_6, \\ \Gamma_9' &= Pe \left[ \Gamma_7 \Gamma_9 + (\Gamma_8 + \delta_1) \Gamma_7' \right] - Lb \Gamma_1 \Gamma_9 \end{aligned} \right\} \quad (18)$$

with conditions

$$\left. \begin{aligned} \Gamma_1 &= 1 + \alpha \Gamma_2 + \beta \Gamma_3, \Gamma_5 = Bi (\Gamma_4 - 1), \\ Nb \Gamma_5 + Nt \Gamma_7 &= 0, \Gamma_8 = 1 \quad \text{at } \eta = 0 \\ \Gamma_2 &\rightarrow 0, \Gamma_4 \rightarrow 0, \Gamma_6 \rightarrow 0, \\ \Gamma_8 &\rightarrow 0 \quad \text{at } \eta \rightarrow \infty \end{aligned} \right\} \quad (19)$$

### 4. Results and Discussion

Boundary conditions (13) are used to solve equations (9-12) using MATLAB's `bvp4c` solver, and the results are graphically displayed for the velocity profile, temperature, concentration, and motile microorganism density profile. The numerical solution arrived at converged on the boundary restrictions without any errors or warnings. This indicates that the established tolerance limit is supported by the numerical solution.

Unless otherwise specified, the emerging parameter values are taken as

$$\begin{aligned} n &= 0.3, Wi = 0.3, Rd = 0.8, M = 0.2, Pr = 0.71, \alpha = 1, \\ Nb &= 0.5, Rb = 0.1, Nt = 0.5, Le = 5, Bi = 2, \lambda = 0.1, \\ Nr &= 0.1, Ec = 0.1, \sigma^* = 0.1, Pe = 1, \delta_1 = 0.1, Lb = 0.2, \\ \beta &= -1 \end{aligned}$$

Figure 2-7 shows the effects of  $M, Wi, n, Rd, Pr$  and  $\lambda$ . Figure 2 shows  $M$  decreases the velocity.

It is obvious that for bigger values of  $M$ , the velocity will decline due to the increase in the Lorentz force.

Figure 3 shows  $Wi$  decreases the velocity.  $Wi$  is the ratio of the fluid's relaxation time to a certain process time; when the Weissenberg number grows, the fluid's relaxation time also increases. This provides resistance for the fluid particles, which lowers the fluid's velocity.

Figure 4 shows  $n$  decreases the velocity. It becomes evident when the greater value of  $n$  depreciates in the fluid velocity. This property affects the fluid's viscosity, physically. Shear thinning is exhibited by the fluid for small values of  $n$ , shear thickening is seen for greater values of  $n$ , and Newtonian behavior is exhibited when  $n = 1$ . A weaker velocity field results when  $n$  is greater than one because the fluid's velocity declines.

Figure 5 shows  $Rd$  increases the velocity. By improving the thermal radiation parameter, the fluid's thermal conditions are significantly improved. This results in greater fluid volume in the boundary layer as a result of the buoyancy effect, which raises the fluid's velocity.

Figure 6 shows  $Pr$  reduces fluid velocity, whereas Figure 7 shows that  $\lambda$  increases fluid velocity.

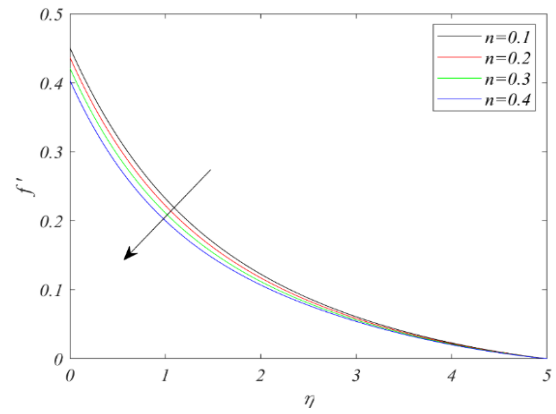


Fig. 4. Variations of  $n$  on velocity

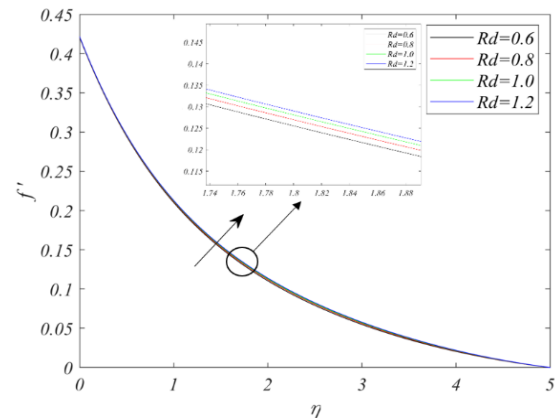


Fig. 5. Variations of  $Rd$  on velocity

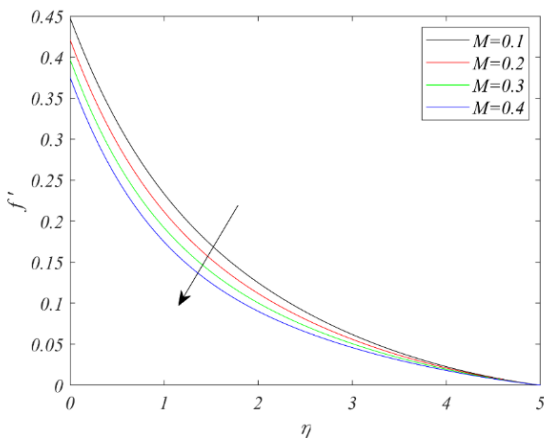


Fig. 2. Variations of  $M$  on velocity

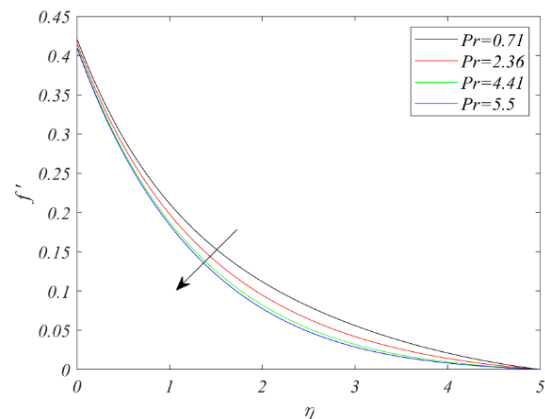


Fig. 6. Variations of  $Pr$  on velocity

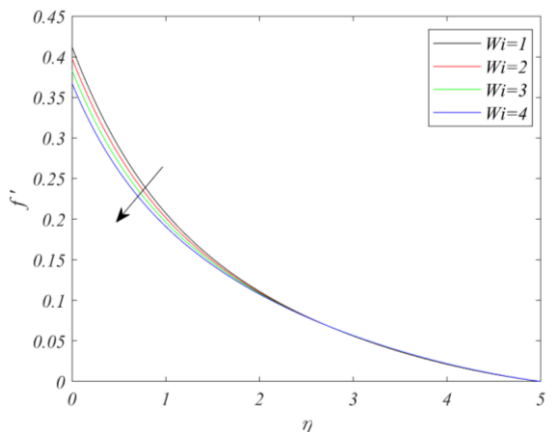


Fig. 3. Variations of  $Wi$  on velocity

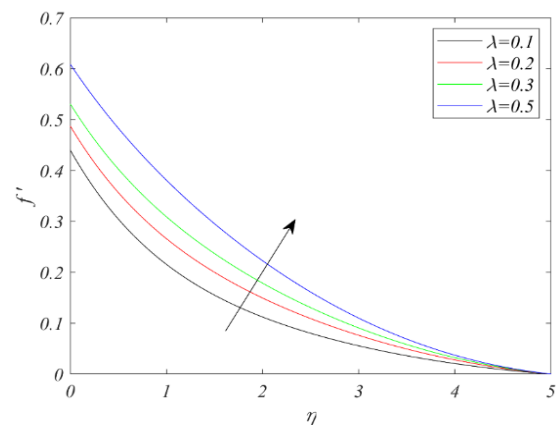


Fig. 7. Variations of  $\lambda$  on velocity

Figure 8-14 displays how temperature is impacted by  $M$ ,  $Wi$ ,  $n$ ,  $Rd$ ,  $Pr$ ,  $Nr$ , and  $Nt$ . The temperature rises in Figure 8 caused by  $M$ . As the magnetic parameter increases, the boundary layer's temperature rises, but the magnitude of its velocity profiles decreases.

The thermal boundary layer thickness has increased, as seen in Figure 9, leading to a rise in temperature.

Figure 10 demonstrates that fluid temperature rises with  $n$ , since it directly influences the energy equation, and it is obvious that the power law index has an impact on temperature distribution. On the other hand, the power law index has a negligible and small impact on the temperature distribution because of its indirect influence on the concentration field.

The temperature is raised by  $Rd$ , as seen in Figure 11. Heat is transported from the heated wall to the fluid due to the fluid's ability to absorb its own radiation.

Figure 12 demonstrates how  $Pr$  lowers the temperature. As the Prandtl number increases, the thermal boundary layer thickness decreases.  $Pr$  is used to describe the relationship between momentum and thermal diffusivity.  $Pr$  controls the relative thickness of the momentum and thermal boundary layers.

Figure 13 depicts a temperature increase caused by  $Nr$ .

$Nt$  causes the temperature to rise, as shown in Figure 14. This is because particles near hot surfaces produce a thermophoretic force that promotes particle breakdown outside of the fluid regime (on the cylinder), increasing temperature and concentration boundary layer thicknesses.

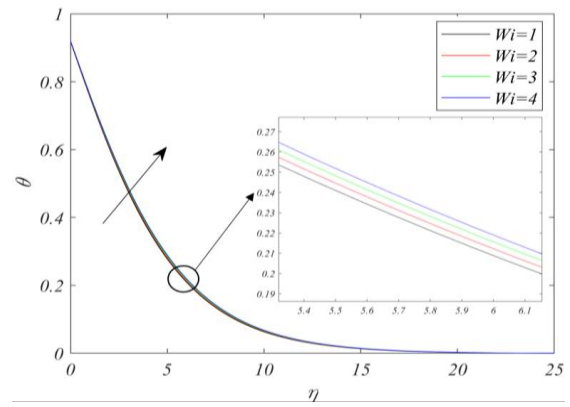


Fig. 9. Variations of  $Wi$  on temperature

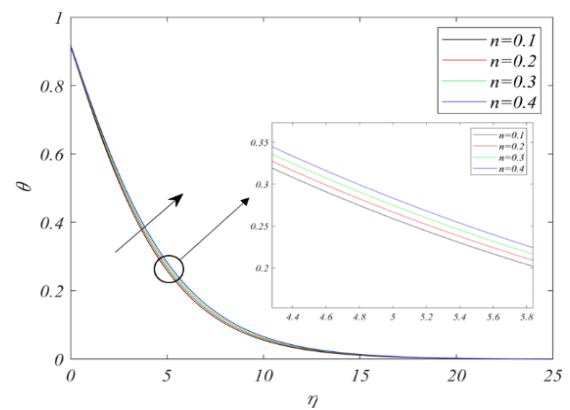


Fig. 10. Variations of  $n$  on temperature profile

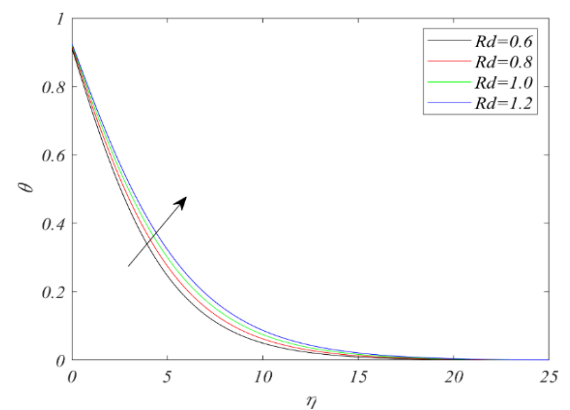


Fig. 11. Variations of  $Rd$  on temperature

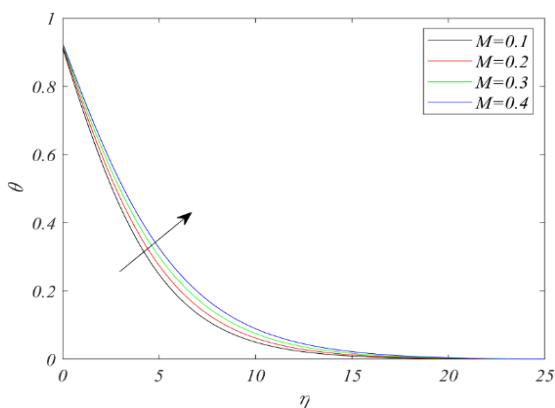


Fig. 8. Variations of  $M$  on temperature

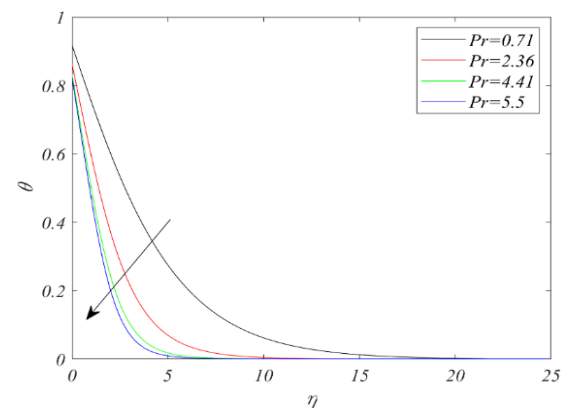


Fig. 12. Variations of  $Pr$  on temperature

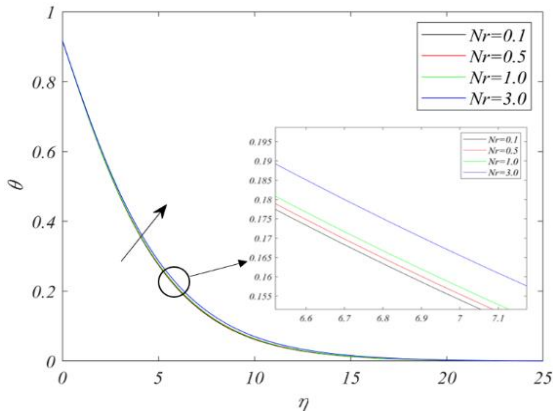


Fig. 13. Variations of  $Nr$  on temperature

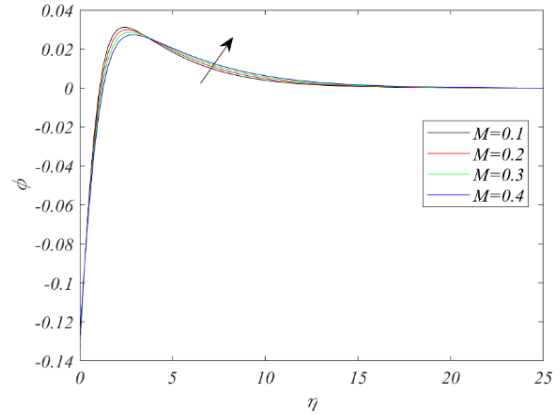


Fig. 15. Variations of  $M$  on concentration

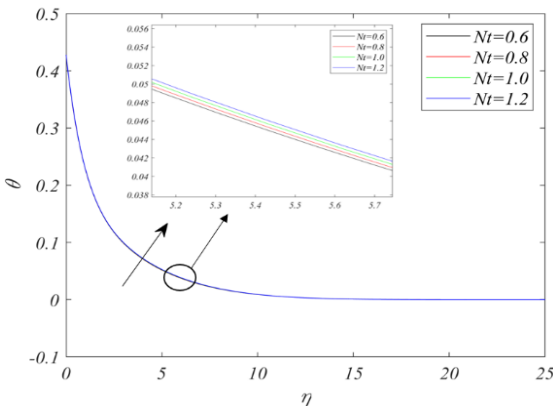


Fig. 14. Variations of  $Nt$  on temperature

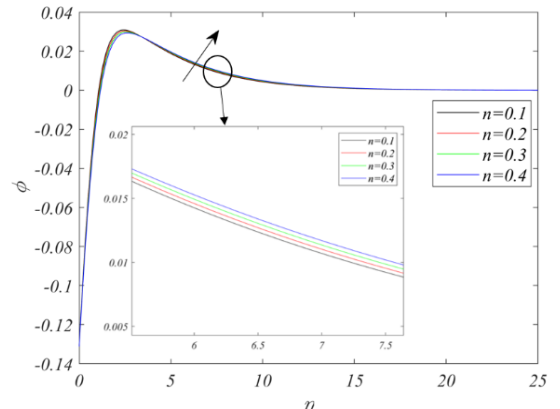


Fig. 16. Variations of  $n$  on concentration

Figure 15-20 shows the effects of  $M$ ,  $n$ ,  $Pr$ ,  $Le$ ,  $Nb$  and  $Nt$  on the field of concentration.

Figure 15 shows that  $M$  increases the concentration field.

Figure 16 shows that  $n$  increases the concentration field; an enhancement in the value of  $n$ , the fluid becomes more viscous, resulting in a concentration field improvement.

Figure 17 shows that  $Pr$  decreases the concentration field.

Figure 18 shows that  $Le$  decreases the concentration field. In the boundary layer, the Lewis number identifies the proportion of thermal diffusion rate to species diffusion rate. According to the definition,  $Le$  is the ratio of the Schmidt to the Prandtl numbers; as a result, Heat will diffuse more quickly than species when  $Le$  is greater than 1.0 and will diffuse at the identical rate when  $Le = 1$ . When the Lewis number is raised, the concentration profiles become steeper and the species boundary layer thins.

Figure 19 shows that  $Nb$  decreases the concentration field due to Brownian motion's ability to warm the boundary layer, which causes the particles to leave the fluid regime.

Figure 20 shows that  $Nt$  increases the concentration field of the fluid.

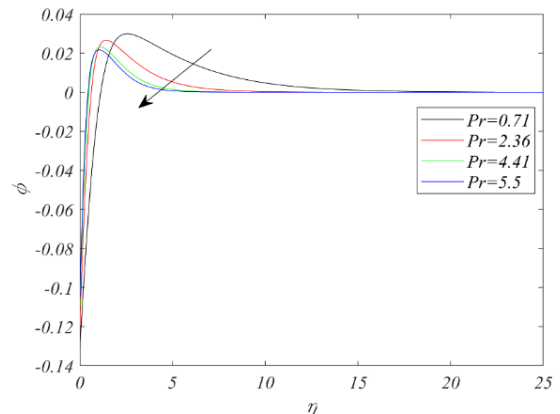


Fig. 17. Variations of  $Pr$  on concentration

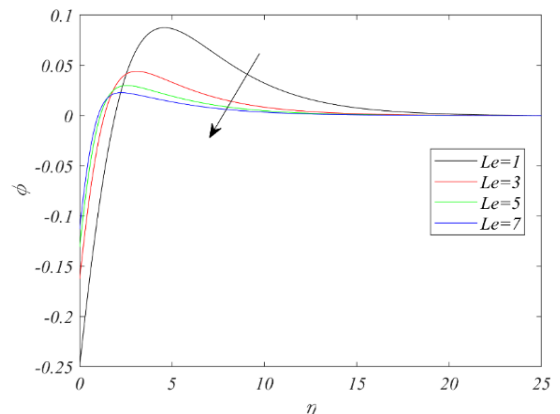


Fig. 18. Variations of  $Le$  on concentration

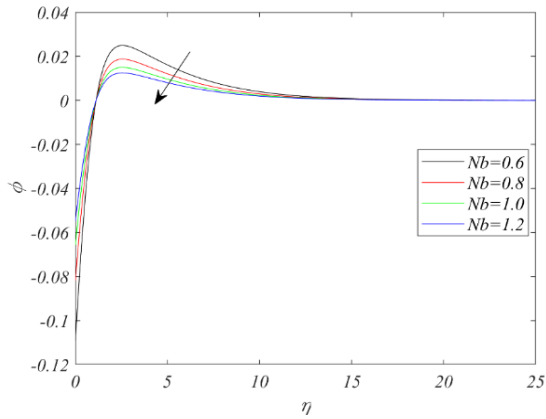


Fig. 19. Variations of  $Nb$  on concentration

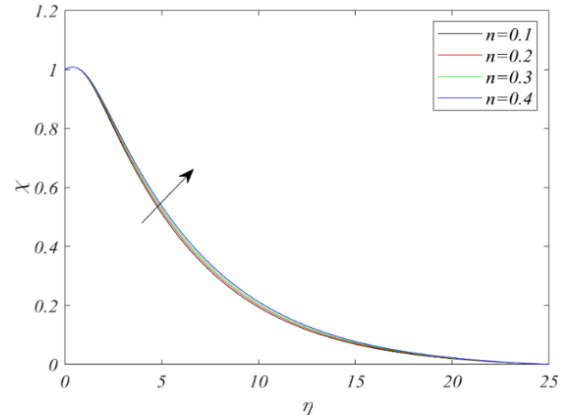


Fig. 21. Variations of  $n$  on motile microorganism density

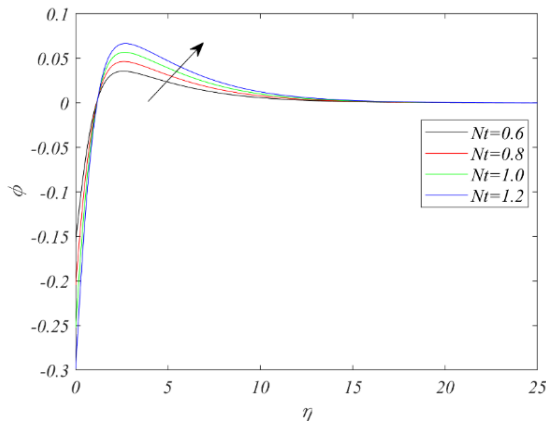


Fig. 20. Variations of  $Nt$  on concentration

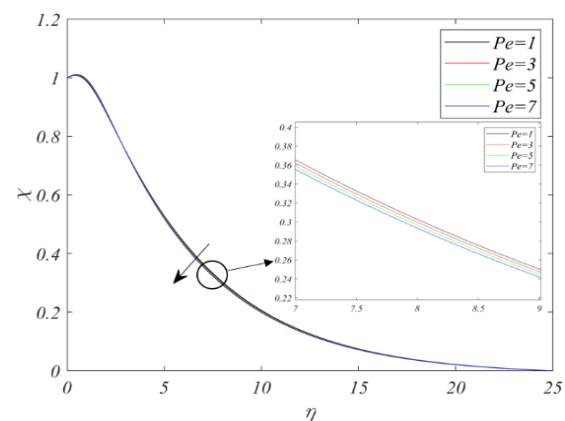


Fig. 22. Variations of  $Pe$  on motile microorganism density

Figures 21-25 show the effects of  $n$ ,  $Pe$ ,  $Lb$ ,  $Nb$  and  $Le$  on motile microorganism density  $\chi$ .

Figure 21 shows that  $n$  increases the motile microorganism density  $\chi$ . An increase in  $n$ , the fluid turns more viscous which improves motile microorganism density.

Figure 22 and Figure 23 show that  $Pe$  and  $Lb$  decrease the motile microorganism density  $\chi$ . A gradual decline in the motile microorganism density of the fluid is observed because the diffusivity of the microorganism decays with increasing Lewis number  $Lb$ .

The thickness of microorganism density decreased in accordance with rising Peclet number  $Pe$  values. With an improvement in Peclet number, the cell swimming speed accelerates. As the number of bio-convection Peclets rises, motile microorganism density degrades. It is seen that the diffusivity of the nanofluid of the motile microorganisms' distribution decreases in both cases.

Figure 24 and Figure 25 show that  $Nb$  and  $Le$  decrease the motile microorganism density  $\chi$ .

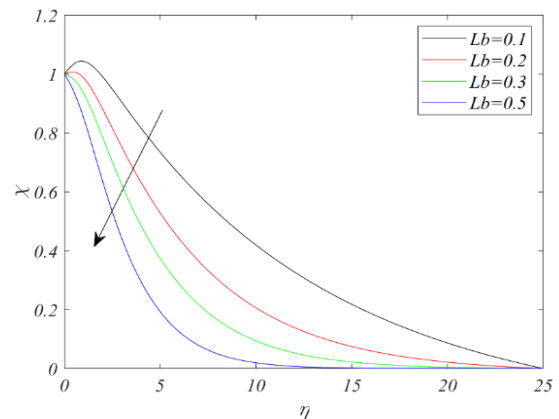


Fig. 23. Variations of  $Lb$  on motile microorganism density

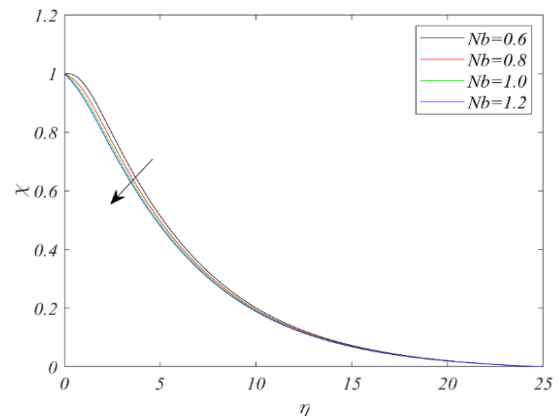


Fig. 24. Variations of  $Nb$  on motile microorganism density



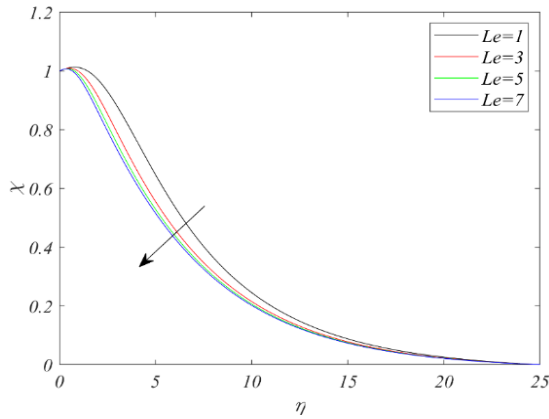


Fig. 25. Variations of  $Le$  on motile microorganism density

Table 1 shows that  $M, Nr, Rb$  increase the Skin friction values, whereas  $\lambda$  shows opposite pattern.

Table 2 shows that  $M, Wi, n, Rd, Nt$  reduce Nusselt number values, whereas  $Pr$  shows the opposite pattern.

Table 3 shows that microorganism density is boosted by increasing the  $Lb$  values and declining the  $Pe$  values.

Table 1. Variations of  $-f''(0)$  for distinct parameters.

$M$	$Nr$	$Rb$	$\lambda$	$-f''(0)$
0.2	0.1	0.1	0.1	0.3098
0.3	0.1	0.1	0.1	0.3116
0.4	0.1	0.1	0.1	0.3127
0.2	0.1	0.1	0.1	0.3098
0.2	0.2	0.1	0.1	0.3101
0.2	0.3	0.1	0.1	0.3105
0.2	0.1	0.1	0.1	0.3098
0.2	0.1	0.2	0.1	0.3132
0.2	0.1	0.3	0.1	0.3167
0.2	0.1	0.1	0.1	0.3098
0.2	0.1	0.1	0.2	0.2952
0.2	0.1	0.1	0.3	0.2846

Table 2. Variations of  $-\theta'(0)$  for different parameters.

$M$	$Wi$	$n$	$Rd$	$Nt$	$Pr$	$-\theta'(0)$
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.3	0.3	0.3	0.8	0.5	0.71	0.2180
0.4	0.3	0.3	0.8	0.5	0.71	0.2144
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.2	0.4	0.3	0.8	0.5	0.71	0.2220

0.2	0.5	0.3	0.8	0.5	0.71	0.2219
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.2	0.3	0.4	0.8	0.5	0.71	0.2199
0.2	0.3	0.5	0.8	0.5	0.71	0.2175
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.2	0.3	0.3	0.9	0.5	0.71	0.2198
0.2	0.3	0.3	1.0	0.5	0.71	0.2177
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.2	0.3	0.3	0.8	0.7	0.71	0.2186
0.2	0.3	0.3	0.8	0.9	0.71	0.2150
0.2	0.3	0.3	0.8	0.5	0.71	0.2222
0.2	0.3	0.3	0.8	0.5	0.81	0.2272
0.2	0.3	0.3	0.8	0.5	0.95	0.2342

Table 3. Variations of  $-\chi'(0)$  for different parameters.

$Pe$	$Lb$	$-\chi'(0)$
1	0.2	0.0361
2	0.2	0.0184
3	0.2	0.0045
1	0.2	0.0361
1	0.3	0.0557
1	0.4	0.0759

### 5. Validity and Comparisons

To prove the validity of the current study, the findings are contrasted with previously published results provided by Ibrahim [8], Khan et al. [9] and Waqas et al. [26].

Table 4 demonstrates that the present results are identical to those of Ibrahim [8], Khan et al. [9] and Waqas et al. [26], and are found in good agreement.

Table 4. Comparison table

$M$	Khan et al. [9]	Waqas et al. [26]	Ibrahim [8]	Present result
0	-1.0000	-1.0000	-1.0000	-1.0000
0.25	---	-1.1180	-1.1180	-1.1180
1.0	-1.4139	-1.4142	-1.4142	-1.4142
5.0	-2.4499	-2.4495	-2.4495	-2.4495
10	-3.3170	-3.3166	-3.3166	-3.3166
100	-10.0503	-10.0499	-10.0499	-10.0499

## 6. Conclusions

The topic of the ongoing study is the tangent hyperbolic nanofluid flow caused by moving microorganisms and second-order velocity slip. A study is done on how Brownian motion and thermophoresis affect fluid.

Using the MATLAB- bvp4c scheme, the remodeled ODEs system is numerically handled. The following outcome has been found

- $Rd$  and  $\lambda$  enhanced the fluid velocity while  $M$ ,  $Wi$  and  $Pr$  show opposite patterns.
- $M$ ,  $Wi$ ,  $n$ ,  $Rd$ ,  $Nr$ , and  $Nt$  enhanced the temperature profile, while  $Pr$  shows the opposite pattern.
- $M$ ,  $n$ , and  $Nt$  to grow Concentration profile, whereas it shows the opposite pattern for  $Pr$ ,  $Le$  and  $Nb$ .
- $Pe$ ,  $Lb$ ,  $Nb$ , and  $Le$  decline the motile microorganism density, whereas  $n$  shows the opposite pattern.
- $M$ ,  $Nr$ ,  $Rb$  increase the Skin friction values, whereas  $\lambda$  shows the opposite pattern.
- $M$ ,  $Wi$ ,  $n$ ,  $Rd$ , and  $Nt$  declines Nusselt number values, whereas  $Pr$  shows the opposite pattern.
- Motile microorganism density is boosted by increasing the  $Lb$  values and declining the  $Pe$  values.

## Nomenclature

$u, v$	velocity components ( $m/s$ )
$B_0$	Magnetic field ( $A/m$ )
$U_0$	Velocity ( $m/s$ )
$u_w$	Stretching velocity ( $m/s$ )
$\alpha$	First order slip velocity
$\beta$	Second order slip velocity
$U_{slip}$	Velocity slip ( $m/s$ )
$Pr$	Prandtl number
$C_\infty$	Ambient fluid concentration ( $Mol/m^3$ )
$\lambda$	Mixed convection parameter
$D_B$	Brownian diffusion ( $m^2 s^{-1}$ )
$D_M$	Microorganisms diffusion ( $m^2 s^{-1}$ )
$Nr$	Buoyancy ratio
$D_T$	Thermophoresis diffusion
$Nu_x$	Local Nusselt number

$\sigma$	Electrical conductivity ( $A^2 S^{-3} / (kg.m^2)$ )
$Lb$	Bioconvection Lewis number
$Sh_x$	Sherwood number
$Rd$	Thermal radiation
$Ec$	Eckert number
$Bi$	Biot number
$Rb$	Bioconvection Rayleigh parameter
$Nn_x$	Microorganism Sherwood number
$N_\infty$	Microorganism density ( $kg.m^{-3}$ )
$Le$	Lewis number
$T_\infty$	Temperature (Ambient fluid) ( $K$ )
$Nt$	Thermophoresis parameter
$Nb$	Brownian motion
$Pe$	Peclet number
$\nu$	Kinematic viscosity ( $m^2/s$ )
$Wi$	Weissenberg number
$n$	Power law index
$\delta_1$	Microorganism diffence parameter
$\sigma^*$	Chemical reaction ( $s^{-1}$ )
$W_c$	Cell swimming speed ( $ms^{-1}$ )
$\rho$	Density ( $kg/m^3$ )
$T_w$	Temperature at the surface ( $K$ )
$C_w$	concentration ( $Mol/m^3$ )
$N$	Microorganism density ( $kg.m^{-3}$ )
$N_w$	Surface microorganism density ( $kg.m^{-3}$ )

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## Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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